Intermittency in cosmic ray showers and its possible consequences

Tadeusz Wibig
University of Łódź, Poland

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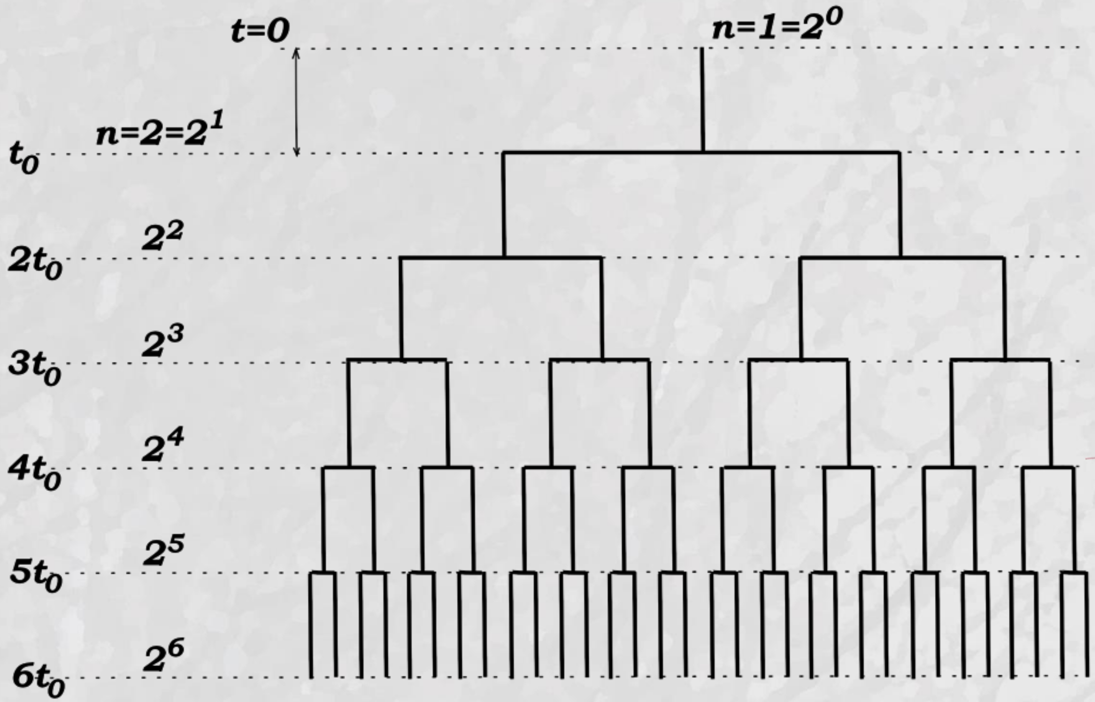
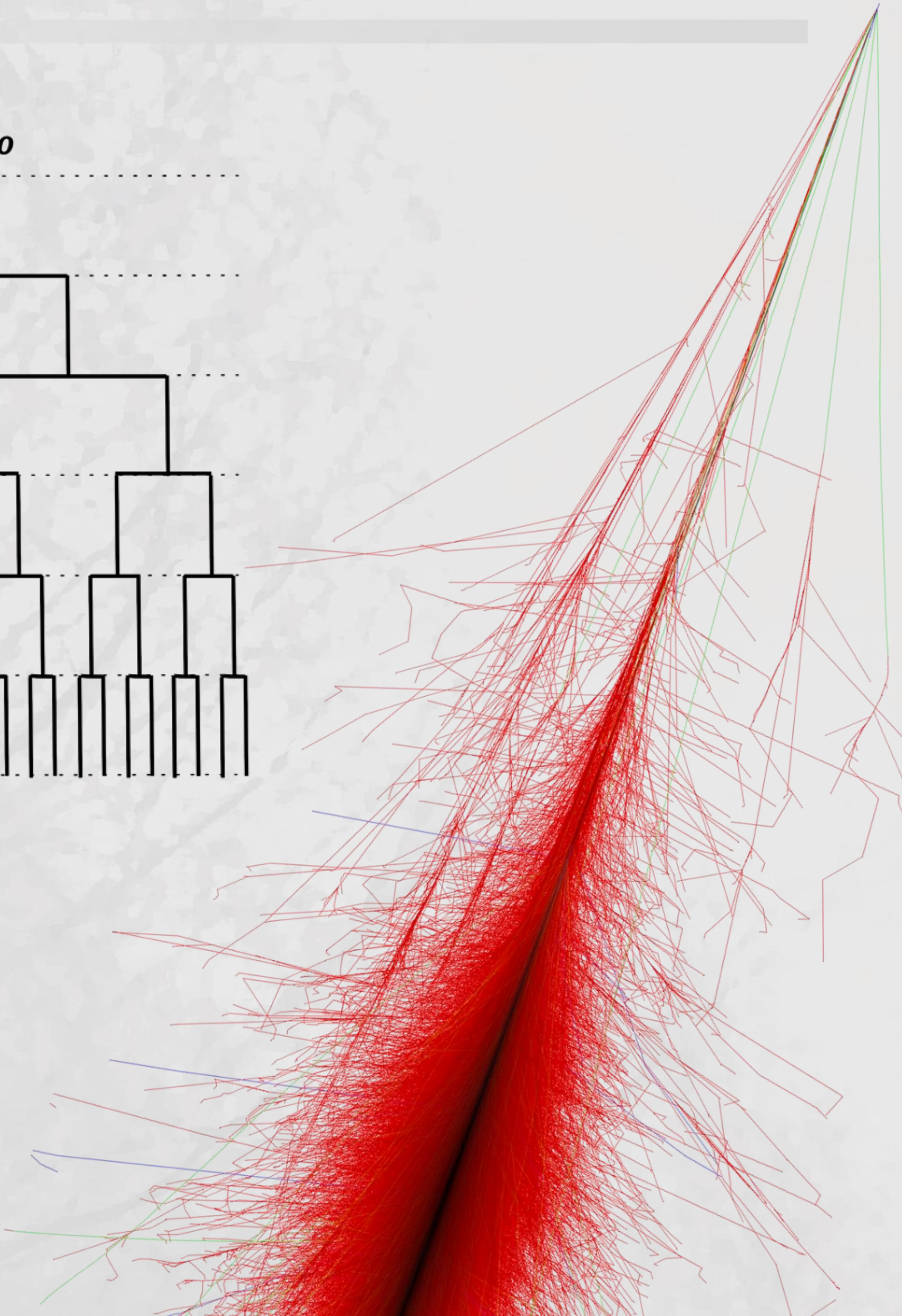

t=0
n=1=2^0
n=2=2^1
t_0
2^2
2t_0
2^3
3t_0
2^4
4t_0
2^5
5t_0
2^6
6t_0



probability ***P*** of registering ***n*** particles in a fixed area detector set at aparticular position $\boldsymbol{x_d}$, $\boldsymbol{y_d}$

$$P\Big|_{N_e, x_c, y_c, \Theta}(n; x_d, y_d)$$

J. F. Carlson and J. R. Oppenheimer, *On multiplicative showers*, Phys. Rev. 51, 220 (1937).

H. J. Bhabha and W. H. Heitler, *The passage of fast electrons and the theory of cosmic showers*, Proceedings of The Royal Society A: Mathematical, Physical and Engineering Sciences 159, 432 (1937).

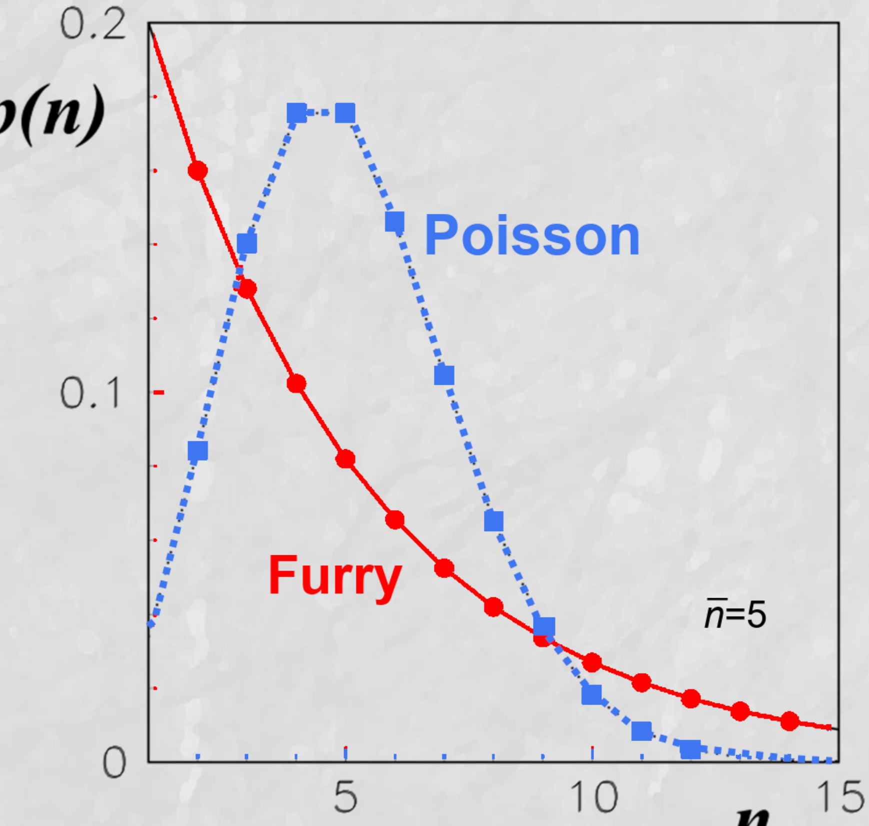
$$P_{\text{Poisson}}(n, \bar{n}) = \frac{\bar{n}^n}{n!}\exp(-\bar{n})$$

$$\left.\frac{\sigma(n)}{\bar{n}}\right|_{\text{Poisson}} = \frac{1}{\sqrt{\bar{n}}} \xrightarrow[\bar{n}\to\infty]{} 0$$

W. H. Furry, *On fluctuation phenomena in the passage of high energy electrons through lead,* Phys. Rev. 52, 569 (1937).

$$P_{\text{Furry}}(n,\bar{n}) = \frac{1}{\bar{n}}\left[1 - 1/\bar{n}\right]^{n}$$

$$\left.\frac{\sigma(n)}{\bar{n}}\right|_{\text{Furry}} = \sqrt{1 - \frac{1}{\bar{n}}} \xrightarrow[\bar{n}\to\infty]{} 1$$



H. Euler*, Die erzeugung hoffmaunscher st¨ose durch multiplikation*, Z. Physik 110, 450 (1938).

Euler found a compromise between these two apparently extreme cases. He noted, *with a hint from Heisenberg*, that the number of particles in a cascade fluctuates as a composite of fluctuating successively participating generations, each of which, fluctuates in a Poisson-like manner.

$$\frac{\sigma(n)}{n} = \frac{\sigma(N)}{N}\left(1 + \frac{1}{N} + \frac{1}{N^2} + \frac{1}{N^3}\cdots\right)$$

where ***N*** is the average number of generations, σ(***N***) is the width of the distribution of the number of generations

(and it itself has a Poisson distribution $\sigma(N)/N = 1/\sqrt{N}$)

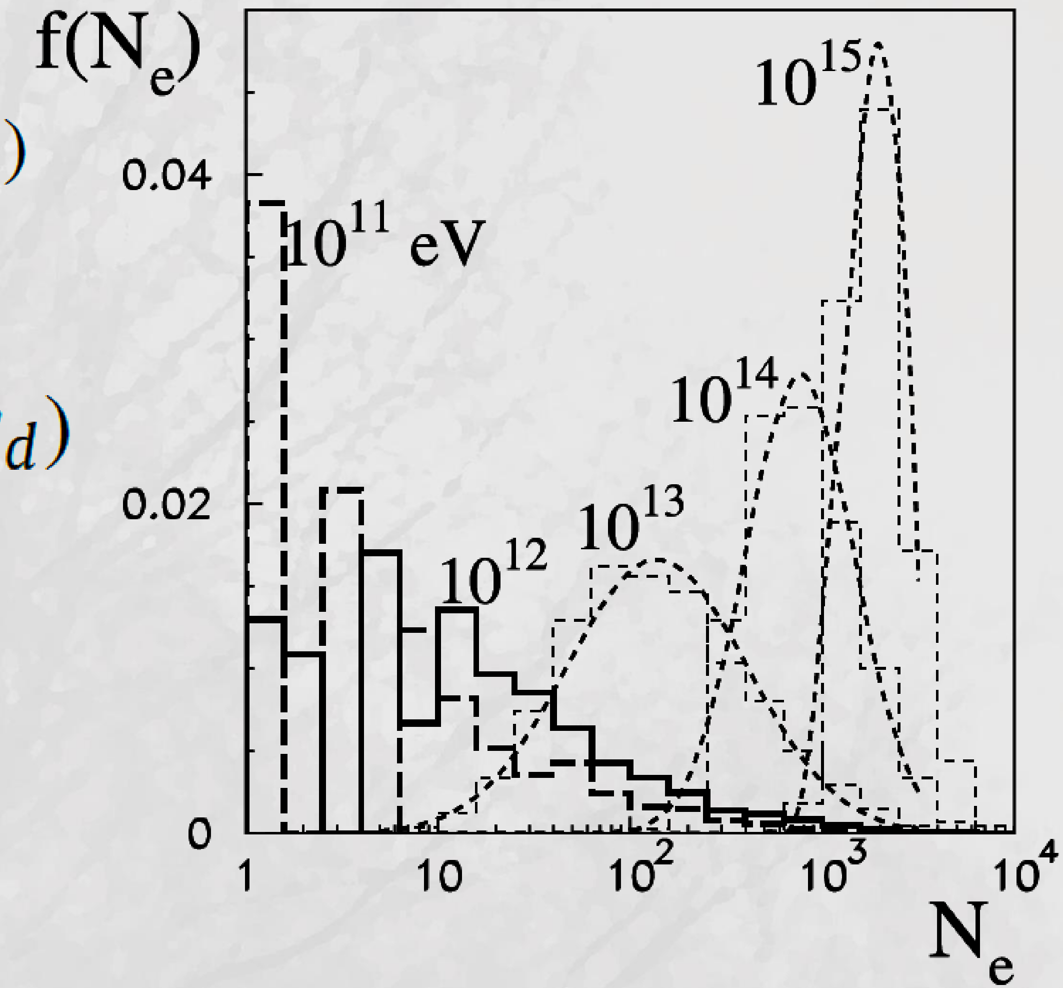
$$\left.\frac{\sigma(n)}{\bar{n}}\right|_{\text{Furry}} \xrightarrow[N\to\infty]{} \left.\frac{\sigma(n)}{\bar{n}}\right|_{\text{Poisson}}$$

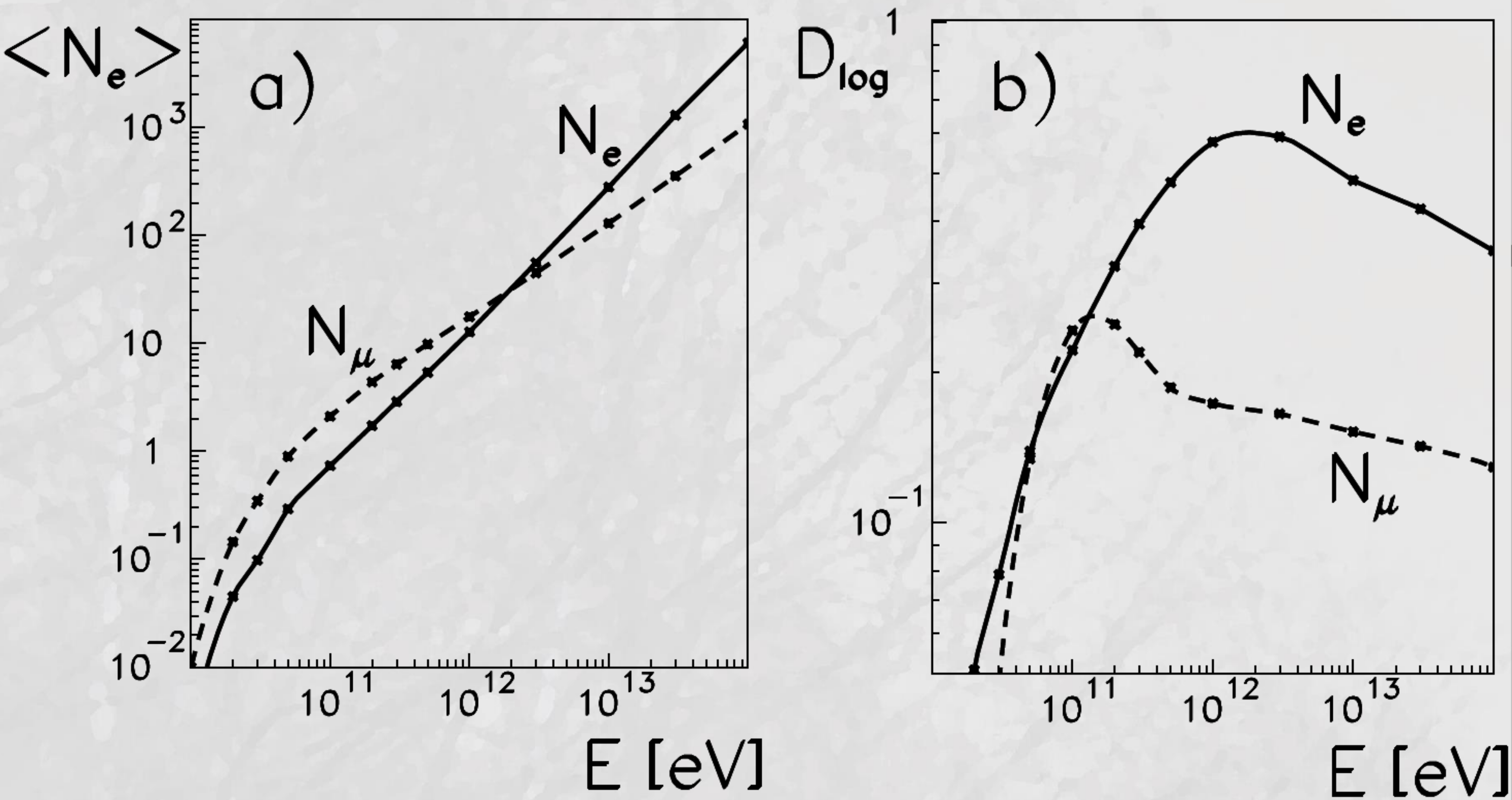
(Furry solutions for thick layers (σ(n)/n = const.) are obtained from the Euler model for ***N*** = 2)

CORSIKA shower electrons with the EPOS-LHC and Gheisha models of hadronic interactions. Lines for E≥10^{13}eV show the log-Normal fits.

$$P\Big|_{N_e, x_c, y_c, \Theta}(n; x_d, y_d)$$

$$P\Big|_{E, x_c, y_c, \Theta}(n; x_d, y_d)$$





<Ne>
a)
Ne
Nμ
10^3
10^2
10
1
10^-1
10^-2
10^11
10^12
10^13
E [eV]
D_log
1
b)
Ne
Nμ
10^-1
10^11
10^12
10^13
E [eV]

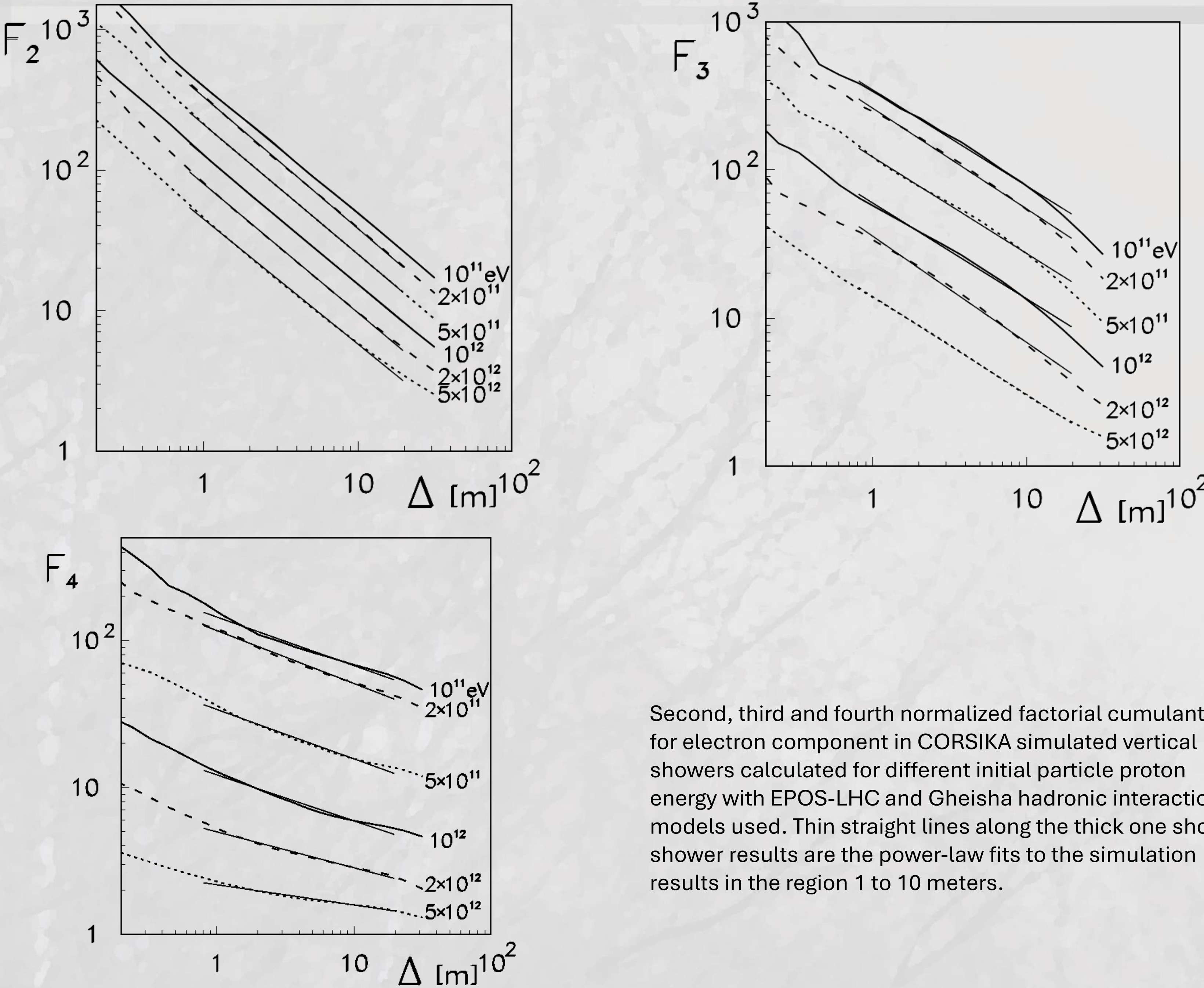
A. Bialas and R. Peschanski *Moments of rapidity distributions as a measure of short-range fluctuations in high-energy collisions*, Nuclear Physics B, 273, 703-718, (1986).
A. Bialas and R. Peschanski *Intermittency in multiparticle production at high energy*, Nuclear Physics B, 308,857-867, (1988).

$$F_2(\Delta) = \frac{\int_{\Omega s} \rho_2(x_1, x_2) dx_1 dx_2}{\int_{\Omega s} \rho(x_1) \rho(x_2) dx_1 dx_2} \qquad d(x_1, x_2) < \Delta$$

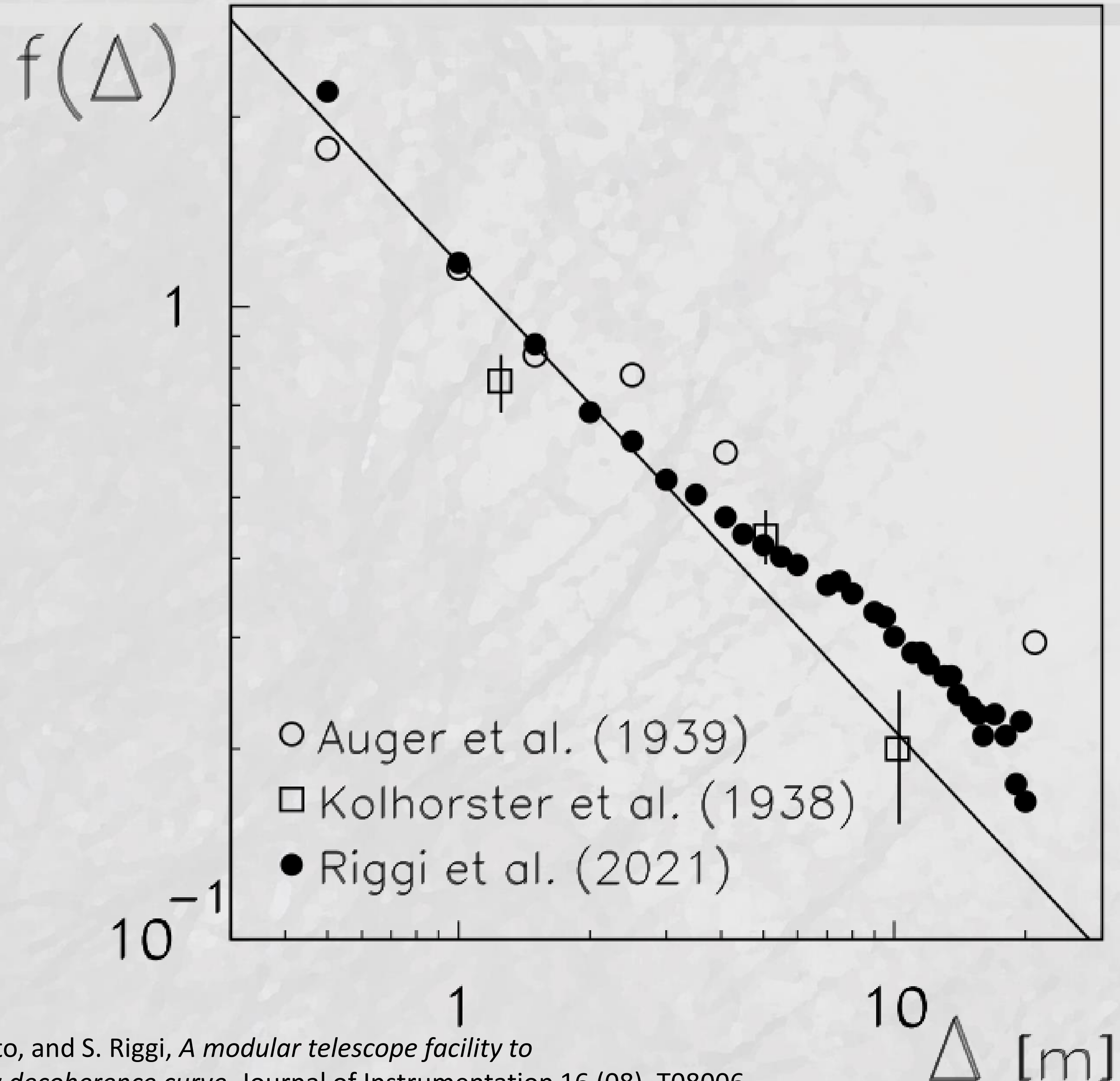
$$F_k(\Delta) = \frac{\int_{\Omega_k} \rho_k(x_1, x_2, ..., x_k)\, dx_1 dx_2 ... dx_k}{\left(\int_{\Omega_k} \rho(x) dx\right)^k}$$

E.A. De Wolf and I.M. Dremin and W. *Kittel Scaling laws for density correlations and fluctuations in multiparticle dynamics*, Physics Reports, 270, 1-141, (1996).
W. Kittel, *Correlations and Fluctuations in High-Energy Collisions, in "Particle Production Spanning MeV and TeV Energies*, pp. 157-182, (2000).

$$F_k(\Delta) = \frac{1}{\text{norm.}}\; k! \sum_{i_1 < i_2 < \ldots < i_k} \prod_{\substack{\text{all pairs} \\ m_1, m_2}} \Theta(\Delta - d(x_{i_{m_1}}, x_{i_{m_2}}))$$



Second, third and fourth normalized factorial cumulants for electron component in CORSIKA simulated vertical showers calculated for different initial particle proton energy with EPOS-LHC and Gheisha hadronic interaction models used. Thin straight lines along the thick one shows shower results are the power-law fits to the simulation results in the region 1 to 10 meters.

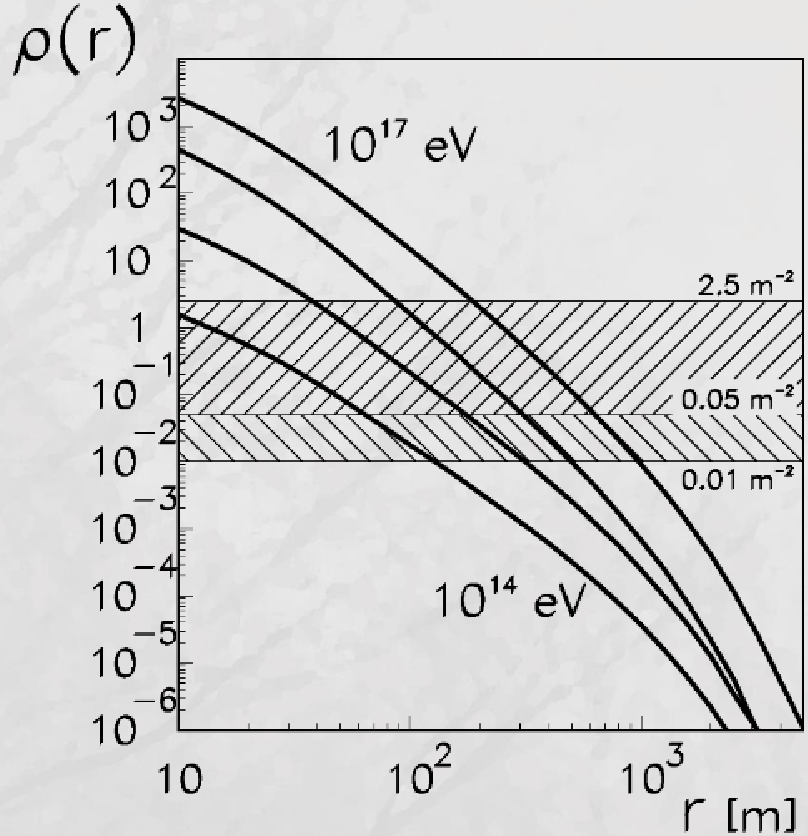


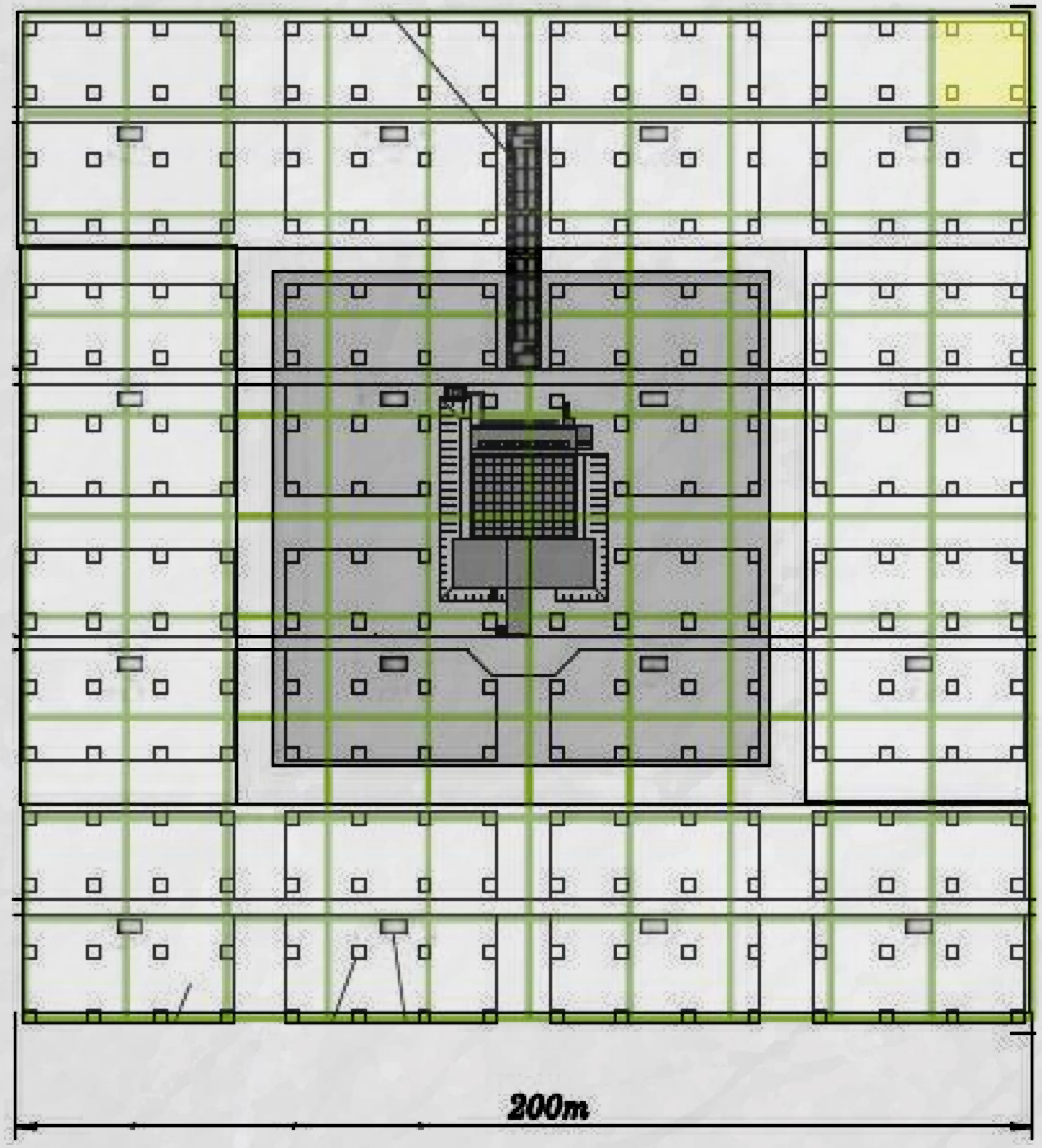
F. Riggi, P. L. Rocca, C. Pinto, and S. Riggi, *A modular telescope facility to investigate the cosmic ray decoherence curve,* Journal of Instrumentation 16 (08), T08006.

P. Auger, P. Ehrenfest, R. Maze, J. Daudin, and R. A. Freon, *Extensive cosmic-ray showers*, Rev. Mod. Phys. 11, 288 (1939).

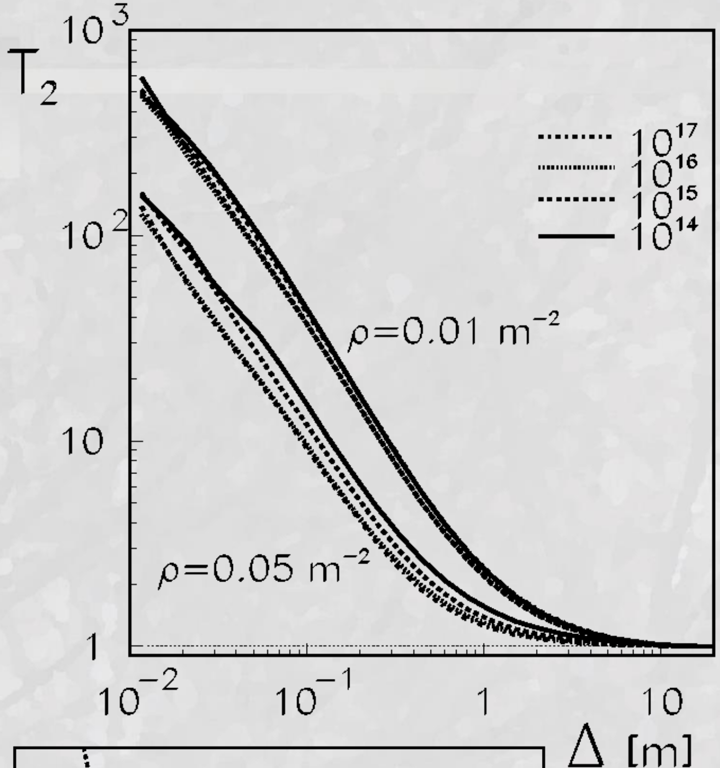
W. E. Kolhorster, I. Matthes, E. Weber, *Gekoppelte Höhenstrahlen*, Naturwissenschaften **26**, 576 (1938)

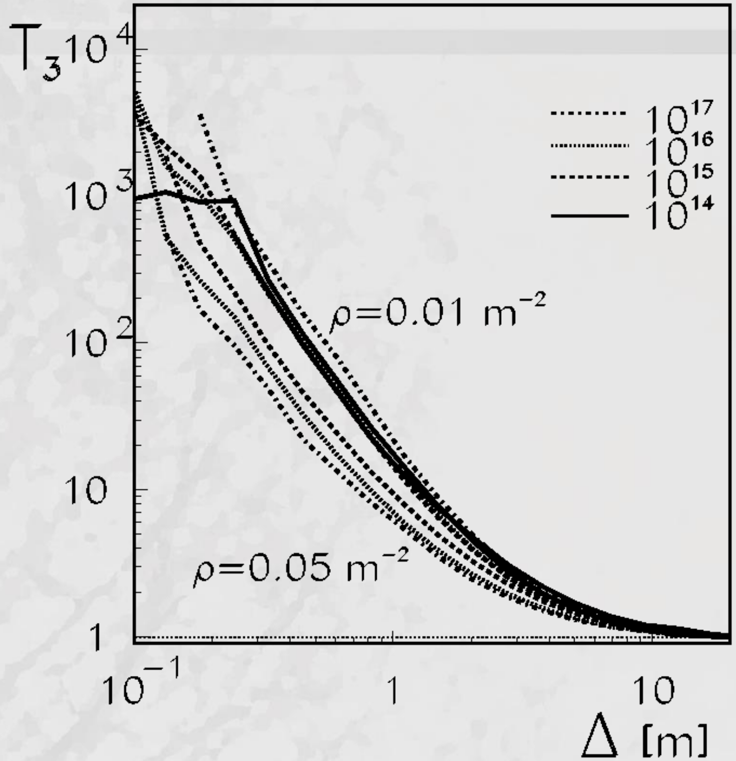

And now for
slightly higher energies

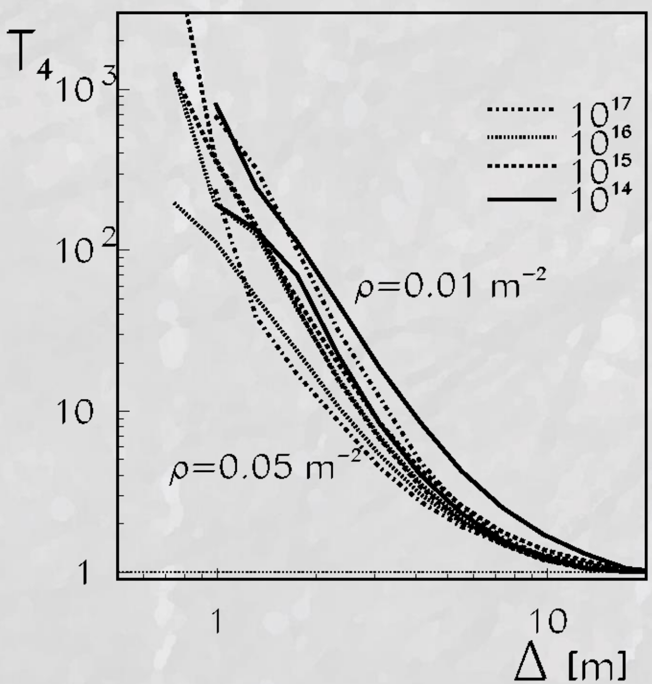

ρ(r)
10^{17} eV
10^{14} eV
2.5 m^{-2}
0.05 m^{-2}
0.01 m^{-2}
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10
10^2
10^3
r [m]



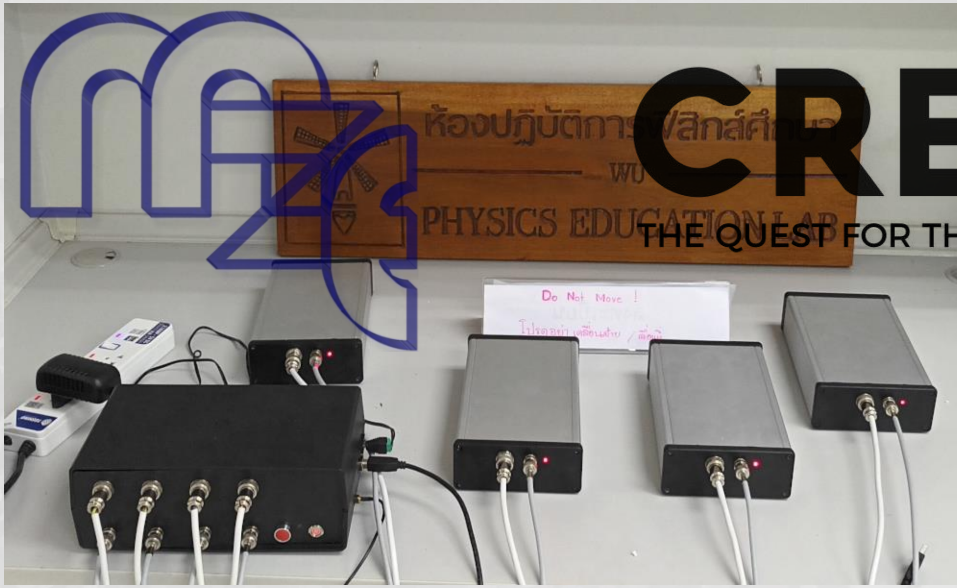
KASCADE



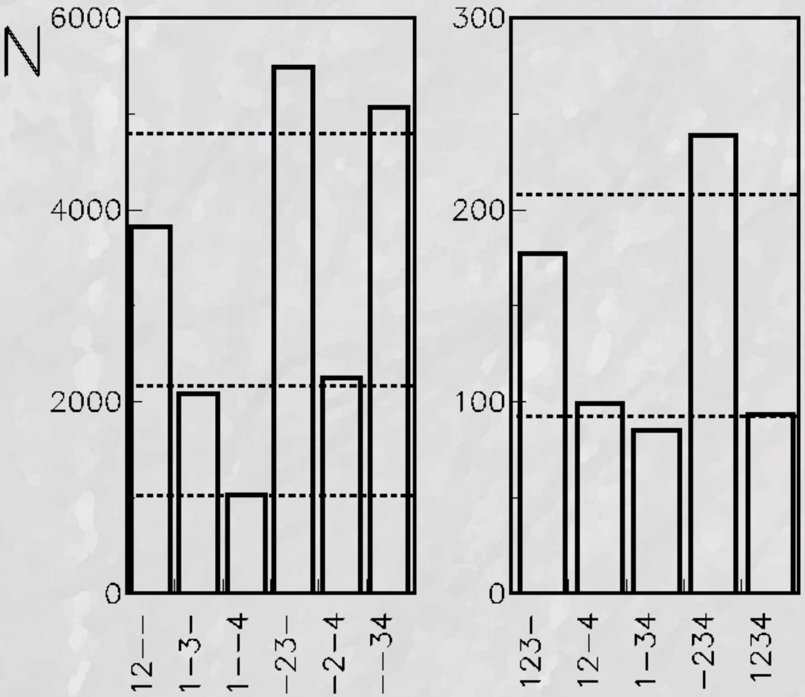


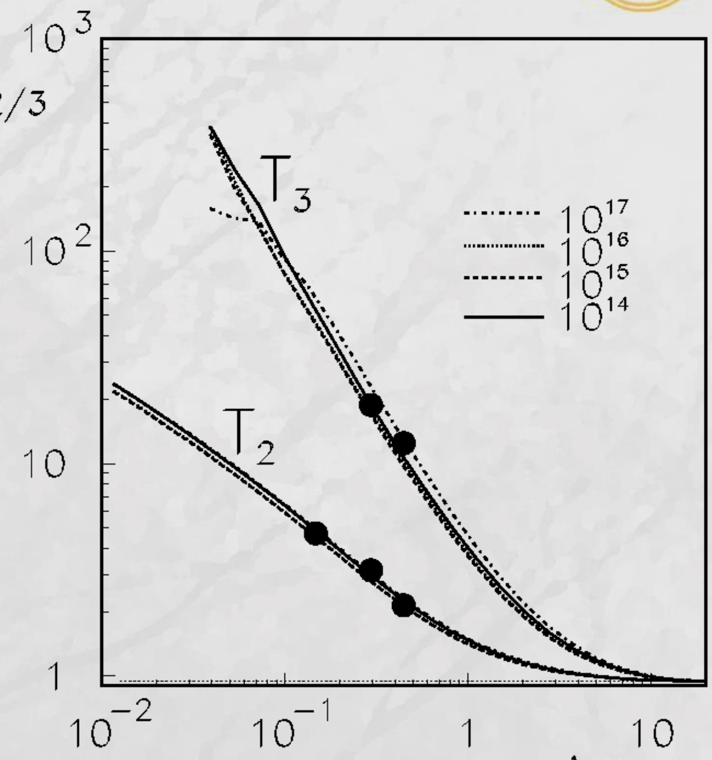


Normalised factor cumulants for the electron component in simulated CORSIKA vertical showers for different initial particle proton energies.
The results are presented for two local particle densities in the shower of 0.05 m^{-2} and 0.01 m^{-2}.

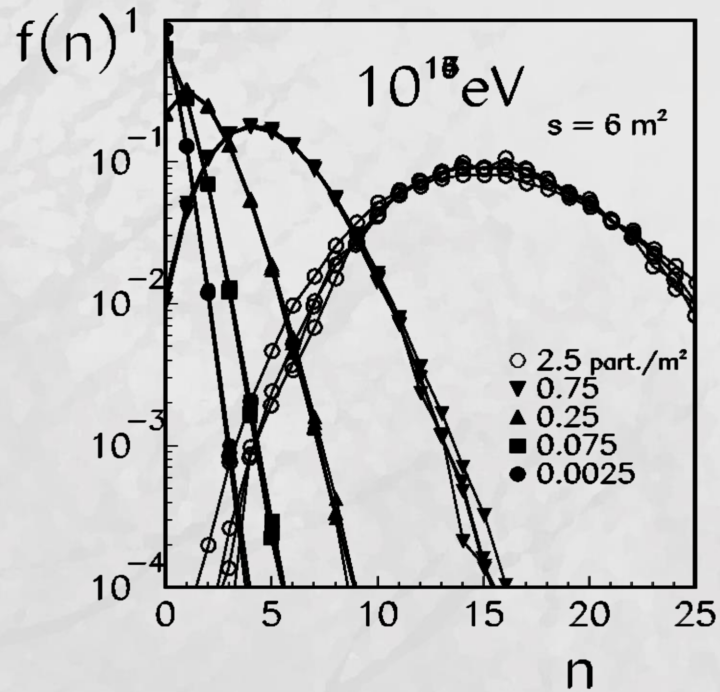


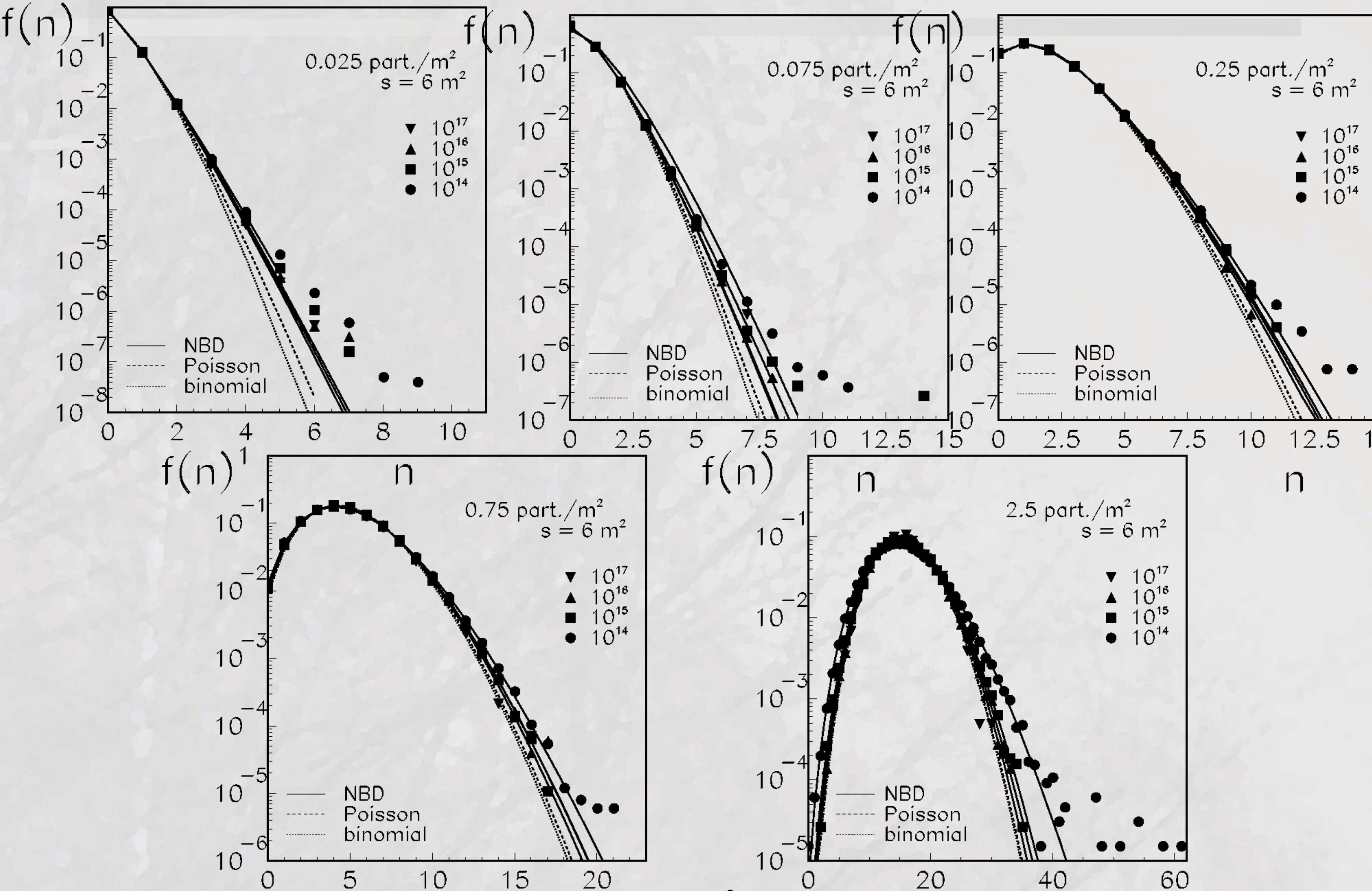
CREDO
THE QUEST FOR THE UNEXPECTED






And now for
slightly higher energies


f(n)
s = 6 m²
2.5 part./m²
0.75
0.25
0.075
0.0025
n

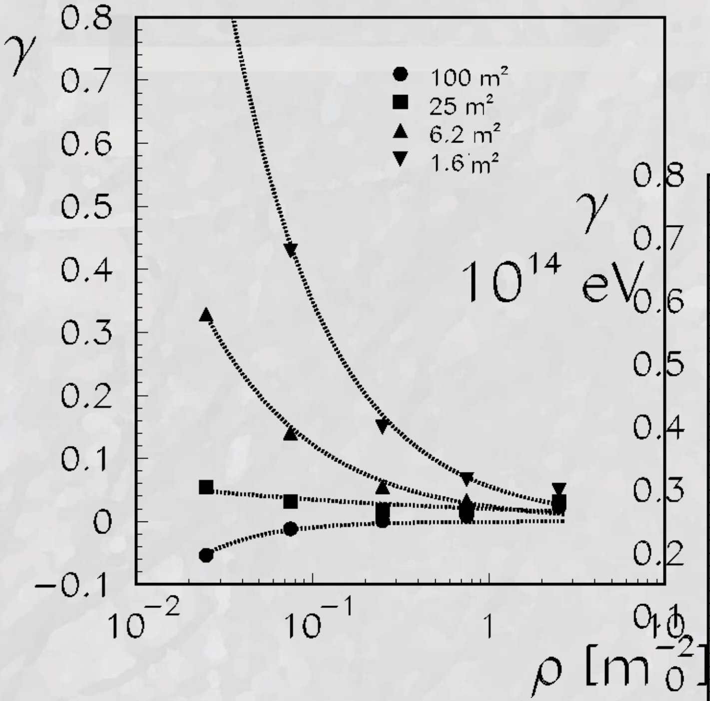


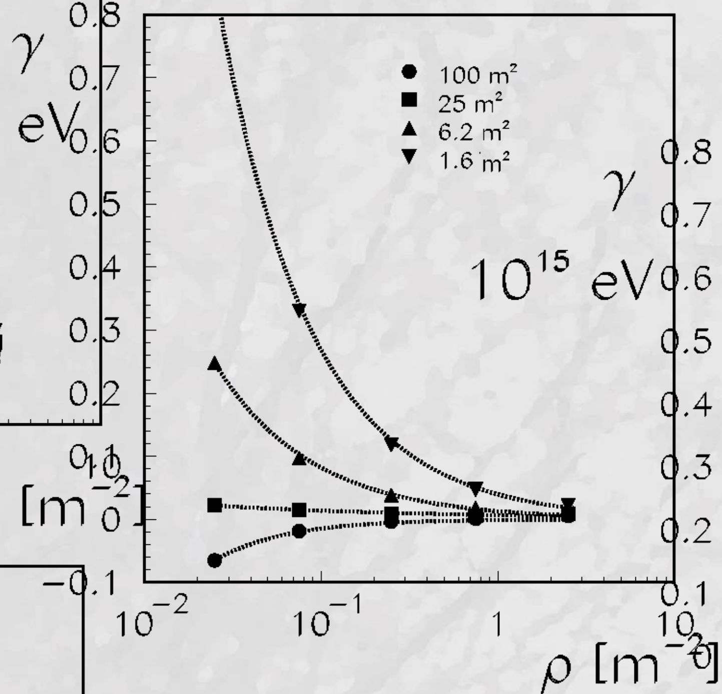
Distributions of particle multiplicities recorded with a ~6 m^2 detector at different expected mean densities defined by the number of particles in a large (400 m^2) quadrant. The lines correspond to a Poisson distribution, a binomial distribution and a series of negative binomial distributions (NBD) fitted to the data for each energy.

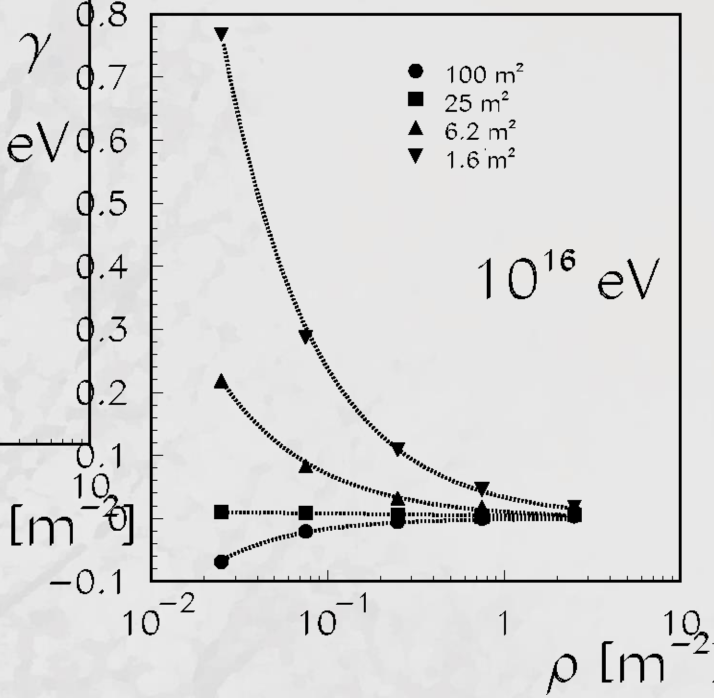
Negative Binomial Distribution (NBD)

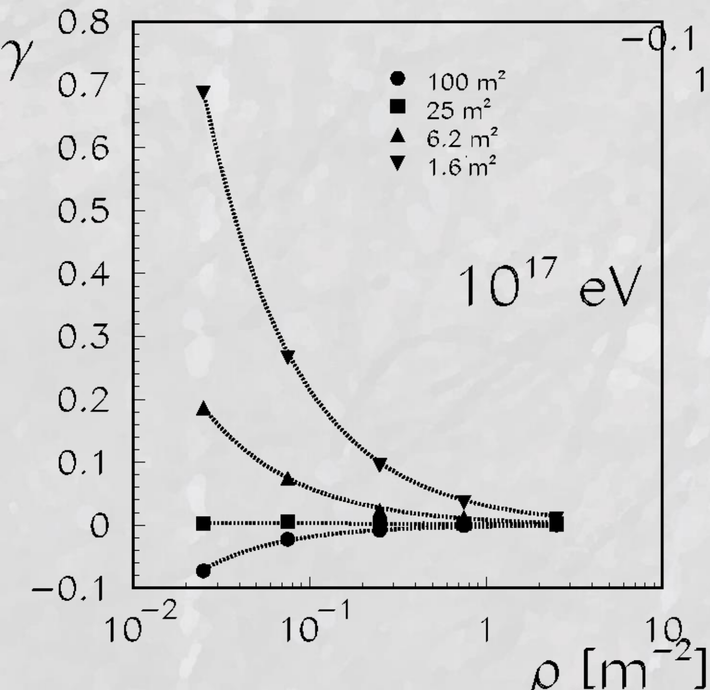
$$p(n, \bar{n}, \gamma) = \binom{n + 1/\gamma - 1}{n} \frac{(\gamma\, \bar{n})^n}{(1 + \gamma\, \bar{n}/)^{(n+1/\gamma)}}$$

$$Var = \bar{n} + \gamma\, \bar{n}^2$$

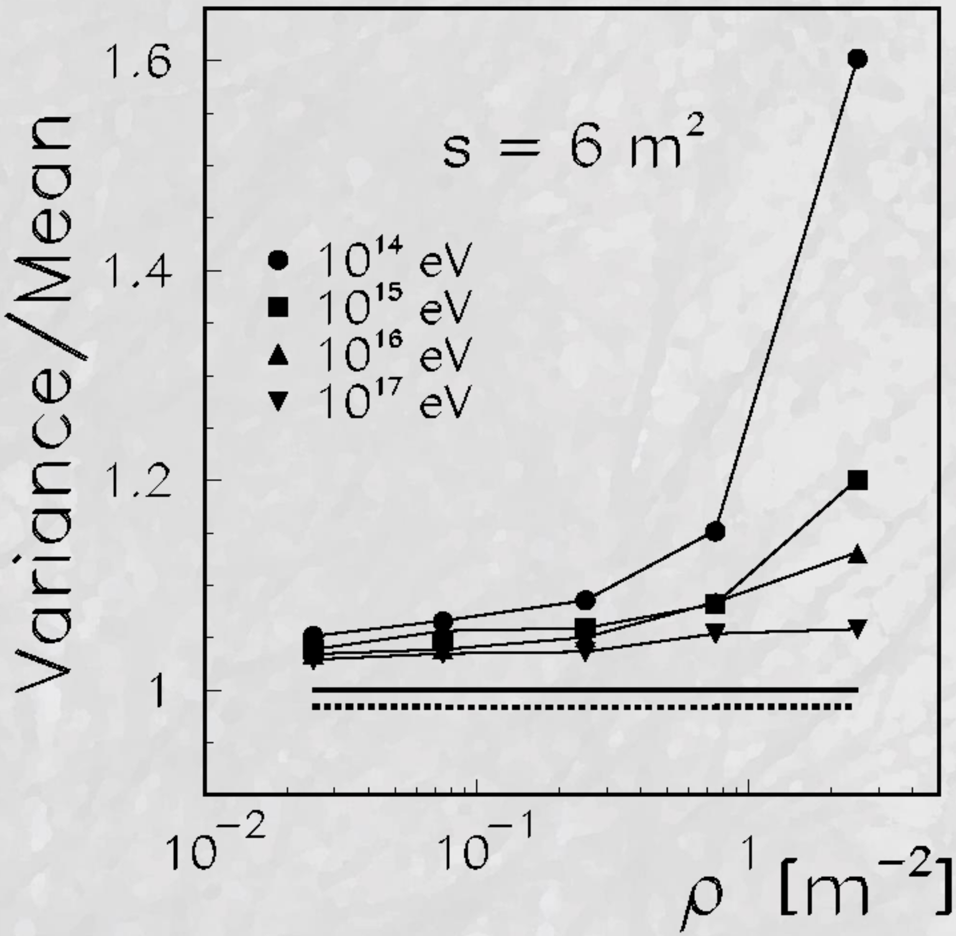








The NBD γ parameter fitted to the simulation results for different primary energies as a function of the expected particle density at the detector position.



Ratio of the variance to the mean for a detector of size ~6 m^2 for different expected local shower densities and for different shower sizes. The thick solid line (with a value of 1 corresponds to the Poisson distribution, and the dashed line corresponds to the corresponding binomial distribution.

$$\mathcal{L} = \prod_{i}^{N_{\text{small}}} f_{\text{Poiss}}(n_i, n(r_i)) \prod_{i}^{N_{\text{large}}} \mathcal{N}(n_i; n(r_i), \sigma_i) \prod_{i}^{N_0} f_0(n(r_i))$$

$$\chi^2 = -2 \ln \mathcal{L}$$

Poisson probability of detecting a small number of particles in the detector, should be modified and replaced by the likelihood for a negative binomial distribution.

Gaussian approximation of the Poisson distribution, also works well for the negative binomial system, but the width of the corresponding Gaussian distribution will be larger than if we use a Poisson distribution ($\gamma \to 0$) with the same mean.

third term gives the probability of not hitting a particular detector at all. The simple assumption that it is Poisson exp(−*n*), when we consider that we are dealing with a negative binomial distribution, should be replaced by $(1 + \gamma\, n)^{-1/\gamma}$.

Negative Binomial Distribution (NBD)

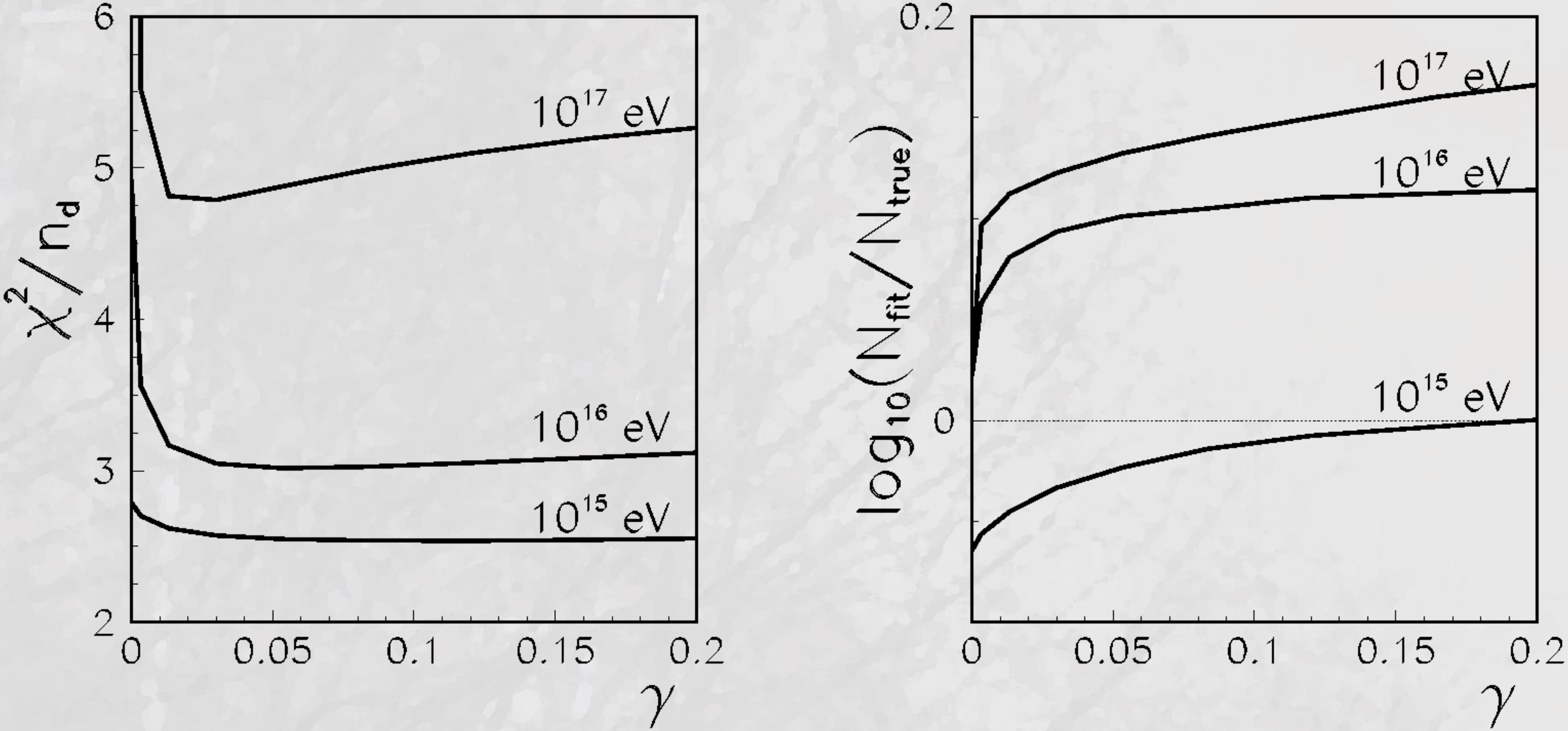
$$p(n,\overline{n},\gamma) = \binom{n + 1/\gamma - 1}{n} \frac{(\gamma\,\overline{n})^n}{(1 + \gamma\,\overline{n}/)^{(n+1/\gamma)}}$$

$$Var = \overline{n} + \gamma\,\overline{n}^2$$

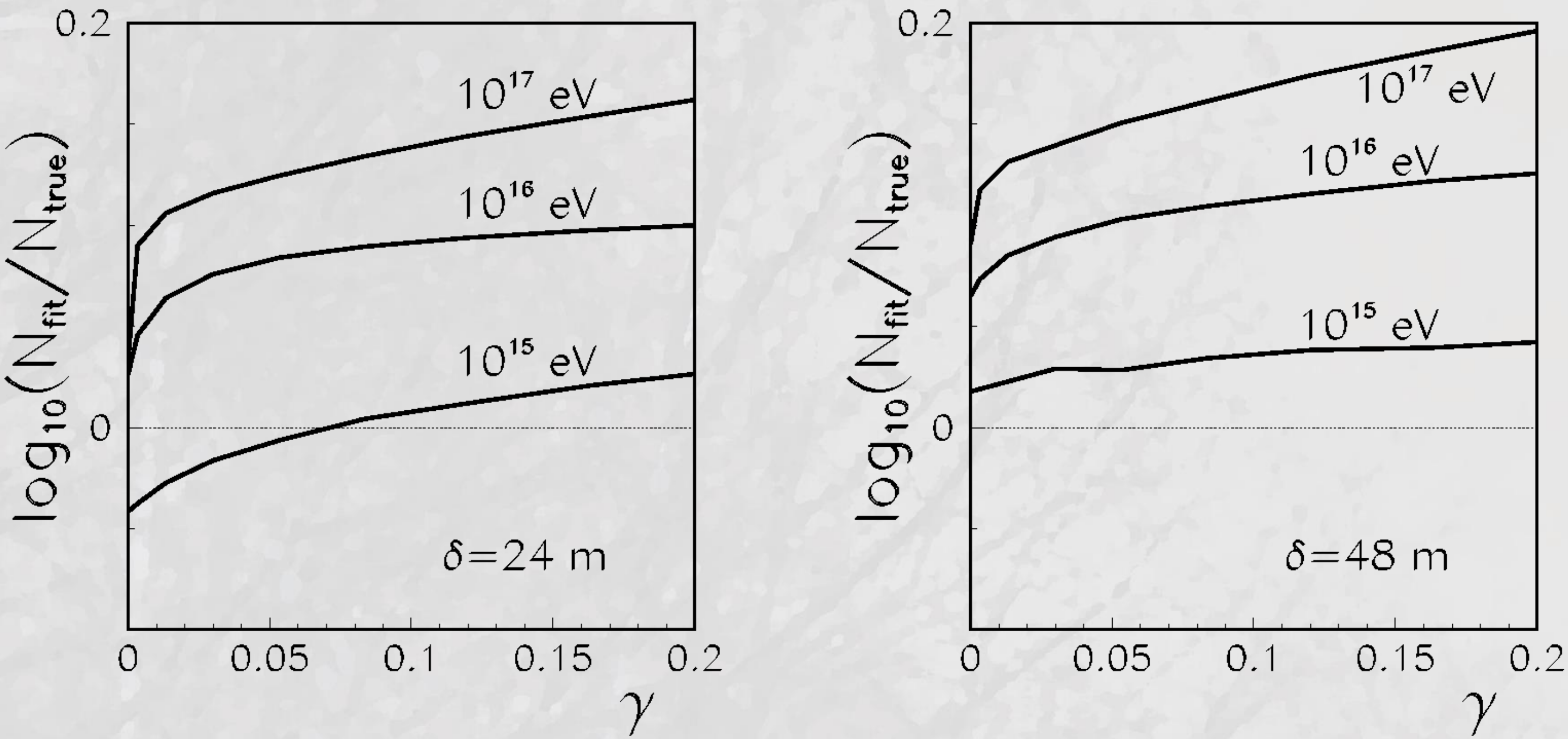
Nishimura–Kamata–Greisen (NKG)

$$\rho_e(r) = \frac{N_e}{2\pi r_0^2} \frac{\Gamma(4.5 - s)}{\Gamma(s)\,\Gamma(4.5 - 2s)} \left(\frac{r}{r_0}\right)^{s-2} \left(1 + \frac{r}{r_0}\right)^{s-4.5},$$

$$s = \frac{3t}{t + 2\ln(E/\epsilon_0)}$$



χ^2/n_d
6
5
4
3
2
0
0.05
0.1
0.15
0.2
γ
10^{17} eV
10^{16} eV
10^{15} eV
$\log_{10}(N_{fit}/N_{true})$
0.2
0
0
0.05
0.1
0.15
0.2
γ
10^{17} eV
10^{16} eV
10^{15} eV



Logarithmic shift of the estimated shower size as a function of the parameter γ for more distant detectors (δ = 24 and 48 m)

Conclusions

- The correlations between the particles are significant for small distances between them.
- These correlations disappear when the distance between the particles is a few metres. They are practically negligible at and above 10 m.
- The kind of the universality of factorial cumulants is seen: their dependence on shower size is very weak.
- The dependence on the mutual distance of the particles is very well described by the power law function.
- The exponent of this power law dependence is constant in the studied range of parameter variability.

- Multiplicity distributions are wider than predicted assuming no correlation and are no longer Poissonian (binomial).

- The negative binomial distribution formula describes these distributions well.

- The relative widths of the multiplicity distributions are significantly wider than expected for Poisson (binomial).

- The consideration of the change in the nature of the particle number distributions in the procedures for the localisation of the axis of extensive air showers leads to corrections in the analysis procedures, the extent of which is, of course, a function of the size of the shower and the geometry and size of the particular shower

- In the CORSIKA simulations, there are - although they are very rare - very significant increases in the number of particles registered by the detector. A simple modification of the density distribution cannot explain them. They require a separate description if they turn out to be important for the interpretation of array data from a particular extensive air shower experiment.

- Simulations were carried out using the geometry of the KASCADE array as an example. The result was that the optimal values of the parameter γ, which is taken as a constant in the localisation procedure, are on the order of 0.05 to 0.1, regardless of the energy of the primary particle, thatis, the size of the shower.

- The resulting optimal values of the χ^2 parameter are significantly smaller than if the Poissonian (gamma = 0) nature of the density fluctuations were assumed.

- There is also a clear bias in the fitted shower size when a wider distribution of density fluctuations is taken intoaccount. The systematic change in the decimal log of the shower size equals approximately 0.05.

- The shower size obtained by assuming Poissonian density fluctuations is underestimated by up to several percent compared to that obtained for NBD (and $\gamma = 0.05$).

- For arrays where we have spaced the detectors further apart, the shower size bias can be greater, up to 20%