

# Decays of the tensor glueball in a chiral approach

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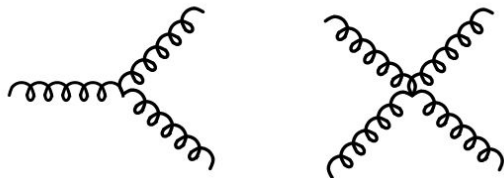
# Overview

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# Introduction

The QCD lagrangian contains gluon self-interaction due to its non-abelian SU(3) symmetry

$$\mathcal{L}_{QCD} = \bar{\psi}_i (i\gamma^\mu (D_\mu)_{ij} - m_i \delta_{ij}) \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$
$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c$$



This begs the question: is there a bound state made of only gluons, a particle that does not contain any matter?

# Bag model

Historically, one of the earliest models in which the glueball mass was computed was the bag model: fields are constricted to a finite volume which has a constant vacuum energy density. Constraint eq:

$$n_\mu G_a^{\mu\nu} = 0$$

The lowest gluon field modes are then transverse electric (TE) and transverse magnetic (TM).

Gluons	$J^{PC}$ Quantum Numbers	Mass
$(\text{TE})^2$	$0^{++}, 2^{++}$	$0.96 \text{ GeV}/c^2$
$(\text{TE})(\text{TM})$	$0^{-+}, 1^{-+}, 2^{-+}$	$1.29 \text{ GeV}/c^2$
$(\text{TE})^3$	$0^{+-}, 1^{++}, 2^{+-}, 3^{++}$	$1.46 \text{ GeV}/c^2$
$(\text{TM})^2$	$0^{++}, 2^{++}$	$1.59 \text{ GeV}/c^2$

One of the most remarkable techniques for glueballs is lattice QCD(LQCD). Spacetime is discretized to a lattice with fermions occupying lattices sites and gauge fields occupying links between sites.

A fit is made between

$$\langle \Omega | \phi^\dagger(t)\phi(0) | \Omega \rangle \propto \int dU \int d\psi \int d\bar{\psi} \sum_{\mathbf{x}} \phi^\dagger(\mathbf{0}, 0)\phi(\mathbf{x}, t) e^{-S_F(\beta) - S_G(\beta)}$$

and

$$\sum_n | \langle \Omega | \phi | n \rangle |^2 \exp(-M_n t)$$

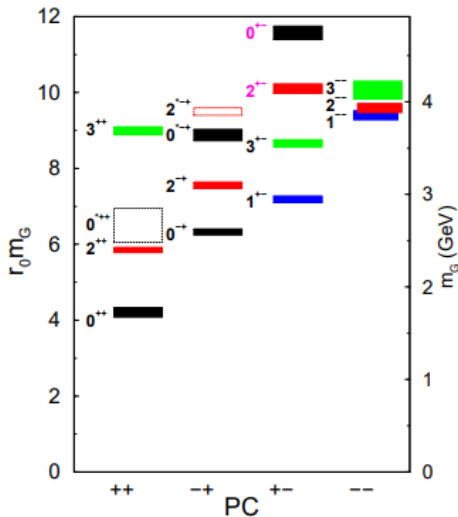
Good for finding states and calculating masses, but in Euclidean space it is complicated to determine decay widths and such.

# Lattice QCD

Lattice calculations have found a large spectrum of pure gluon states.

The tensor ( $J^{PC} = 2^{++}$ ) is the second lightest glueball and so one of the best candidates for experimental verification.

Lattice calculations have some difficulties computing decay rates, so there is room for us to find new information using our chiral model.



Peardon, Morningstar 1999

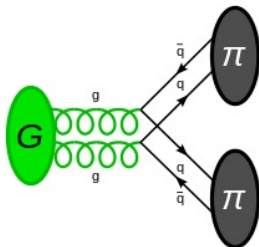
# Glueball width

Glueballs are expected to have relatively small decay widths, from large  $N_c$  scaling:

$$A_{gg \rightarrow \bar{q}q + \bar{q}q} \propto N_c^{-1}$$

$$A_{\bar{q}q \rightarrow \bar{q}q + \bar{q}q} \propto N_c^{-\frac{1}{2}}$$

Similarly, any process glueball  $\rightarrow$  hadrons is suppressed because of the OZI rule



# Experimental Search

Numerous experiments are working on data related to glueballs

- BESIII
- LHCb
- GlueX
- PANDA

Experimentally  $J/\psi$  decays are one of the best places to search for glueballs, like in BESIII data

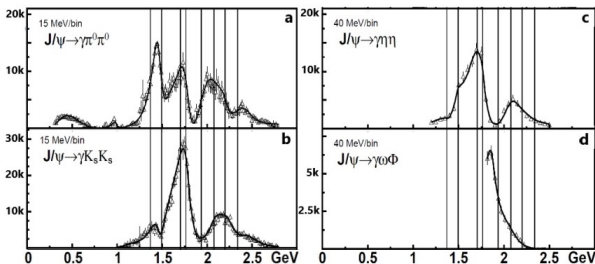


Figure 1: Number of events in the  $S$ -wave as functions of the two-meson invariant mass from the reactions  $J/\psi \rightarrow \gamma \pi^0 \pi^0$  (a),  $K_S K_S$  (b),  $\eta \eta$  (c),  $\phi \omega$  (d). (a) and (b) are based on the analysis of  $1.3 \cdot 10^9$   $J/\psi$  decays, (c) and (d) on  $0.225 \cdot 10^9$   $J/\psi$  decays.



# Linear Sigma Model

The most important symmetry breaking patterns for the eLSM are:

- Breaking of dilatation symmetry by dilaton field  $G$  (scalar glueball), leading to gluon condensate

$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4} \frac{m_G^2}{\Lambda_G^2} \left[ G^4 \log\left(\frac{G}{\Lambda_G}\right) - \frac{G^4}{4} \right]$$

- Spontaneous chiral symmetry breaking; QCD Lagrangian is (almost) invariant under chiral transformations, but the vacuum is not. This leads to a chiral condensate and pions as massless scalars
- The condensates lead to shifts e.g.  $G \rightarrow G + G_0, \Phi \rightarrow \Phi + \Phi_0$  which leads to mass terms similarly to the Higgs mechanism.
- Explicit chiral symmetry breaking gives pions a small mass compared to the other mesons

# Linear Sigma Model

The LSM was previously extended for tensor and axial tensor mesons and its decay products of vectors, axial vectors, etc.

$$\mathcal{L}_{\text{eLSM}} = \mathcal{L}_{\text{dil}} + \text{Tr} \left[ \left( D_\mu \Phi \right)^\dagger \left( D_\mu \Phi \right) \right] - m_0^2 \left( \frac{G}{G_0} \right)^2 \text{Tr} \left[ \Phi^\dagger \Phi \right] - \frac{1}{4} \text{Tr} \left[ \left( L_{\mu\nu}^2 + R_{\mu\nu}^2 \right) \right] + \dots ,$$

Gave us decent results for tensor mesons:

Decay process (in model)	eLSM (MeV)	PDG-2020 (MeV)
$a_2(1320) \rightarrow \rho(770) \pi$	$71.0 \pm 2.6$	$73.61 \pm 3.35 \leftrightarrow (70.1 \pm 2.7)\%$
$K_2^*(1430) \rightarrow K^*(892) \pi$	$27.9 \pm 1.0$	$26.92 \pm 2.14 \leftrightarrow (24.7 \pm 1.6)\%$
$K_2^*(1430) \rightarrow \rho(770) K$	$10.3 \pm 0.4$	$9.48 \pm 0.97 \leftrightarrow (8.7 \pm 0.8)\%$
$K_2^*(1430) \rightarrow \omega(782) K$	$3.5 \pm 0.1$	$3.16 \pm 0.88 \leftrightarrow (2.9 \pm 0.8)\%$
$f_2'(1525) \rightarrow K^*(892) K + \text{c.c.}$	$19.89 \pm 0.73$	

Compared to the work on tensor mesons, we need to replace the tensors to realize flavour blindness:

$$T_{\mu\nu} \longrightarrow G_{2,\mu\nu} \cdot \mathbf{1}$$

The lagrangian leading to tensor glueball decays involves solely left- and right-handed chiral fields:

$$\mathcal{L} = \lambda G_{\mu\nu} \left( \text{Tr} \left[ \{L^\mu, L^\nu\} \right] + \text{Tr} \left[ \{R^\mu, R^\nu\} \right] \right)$$

Left- and right-handed fields are in terms of the vector and axial vector nonets

$$L^\mu := V^\mu + A_1^\mu, \quad R^\mu := V^\mu - A_1^\mu.$$

# Tensor glueball decays

The Lagrangian leads to three kinematically allowed decay channels

- Decaying of the tensor glueball to the two pseudoscalar mesons have the following decay rate formula

$$\Gamma_{G_2 \rightarrow P^{(1)} + P^{(2)}} = \frac{\kappa_{gpp,i} \lambda^2 |\vec{k}_{p^{(1)},p^{(2)}}|^5}{60 \pi m_{G_2}^2};$$

- while for the vector and pseudoscalar mesons

$$\Gamma_{G_2 \rightarrow V^{(1)} + V^{(2)}} = \frac{\kappa_{gVV,i} \lambda^2 |\vec{k}_{V^{(1)},V^{(2)}}|}{120 \pi m_{G_2}^2} \left( 15 + \frac{5 |\vec{k}_{V^{(1)},V^{(2)}}|^2}{m_{V^{(1)}}^2} + \frac{5 |\vec{k}_{V^{(1)},V^{(2)}}|^2}{m_{V^{(2)}}^2} + \frac{2 |\vec{k}_{V^{(1)},V^{(2)}}|^4}{m_{V^{(1)}}^2 m_{V^{(2)}}^2} \right);$$

- and for the axial-vector and pseudoscalar mesons

$$\Gamma_{G_2 \rightarrow A_1 + P} = \frac{\kappa_{gap,i} \lambda^2 |\vec{k}_{a_1,p}|^3}{120 \pi m_{G_2}^2} \left( 5 + \frac{2 |\vec{k}_{a_1,p}|^2}{m_{a_1}^2} \right).$$

# Branching ratios

- $\lambda$  is not known so we can only compute branching ratios
- Computation is done for a tensor glueball mass of 2210 MeV
- Vector channels are dominant, in particular  $\rho\rho$  and  $K^*K^*$
- Serves as a qualitative baseline, we can input different masses when comparing to specific resonances

Branching Ratio	theory
$\frac{G_2(2210) \rightarrow \bar{K} K}{G_2(2210) \rightarrow \pi \pi}$	0.4
$\frac{G_2(2210) \rightarrow \eta \eta}{G_2(2210) \rightarrow \pi \pi}$	0.1
$\frac{G_2(2210) \rightarrow \eta \eta'}{G_2(2210) \rightarrow \pi \pi}$	0.004
$\frac{G_2(2210) \rightarrow \eta' \eta'}{G_2(2210) \rightarrow \pi \pi}$	0.006
$\frac{G_2(2210) \rightarrow \rho(770) \rho(770)}{G_2(2210) \rightarrow \pi \pi}$	55
$\frac{G_2(2210) \rightarrow K^*(892) K^*(892)}{G_2(2210) \rightarrow \pi \pi}$	46
$\frac{G_2(2210) \rightarrow \omega(782) \omega(782)}{G_2(2210) \rightarrow \pi \pi}$	18
$\frac{G_2(2210) \rightarrow \phi(1020) \phi(1020)}{G_2(2210) \rightarrow \pi \pi}$	6
$\frac{G_2(2210) \rightarrow a_1(1260) \pi}{G_2(2210) \rightarrow \pi \pi}$	0.24
$\frac{G_2(2210) \rightarrow K_{1,A} K}{G_2(2210) \rightarrow \pi \pi}$	0.08
$\frac{G_2(2210) \rightarrow f_1(1285) \eta}{G_2(2210) \rightarrow \pi \pi}$	0.02
$\frac{G_2(2210) \rightarrow f_1(1420) \eta}{G_2(2210) \rightarrow \pi \pi}$	0.01

## $f_2(1910)$

- the meson  $f_2(1910)$  has a width of  $167 \pm 21$  MeV and it decays into (among others)  $\eta\eta$  and  $K\bar{K}$ .

- the decay ratio

$$\frac{\rho(770)\rho(770)}{\omega(782)\omega(782)}$$

of about  $2.6 \pm 0.4$  is not far from the theoretical result of 3.1

- this state cannot be mainly gluonic since the experimental ratio  $\eta\eta/\eta\eta'(958)$  is less than 0.05, while the theoretical result is much larger (about 8).

Branching Ratio	eLSM
$\frac{f_2(1910) \rightarrow \rho(770)\rho(770)}{f_2(1910) \rightarrow \pi\pi}$	62
$\frac{f_2(1910) \rightarrow \omega(782)\omega(782)}{f_2(1910) \rightarrow \pi\pi}$	20
$\frac{f_2(1910) \rightarrow \eta\eta}{f_2(1910) \rightarrow \pi\pi}$	0.077
$\frac{f_2(1910) \rightarrow \eta\eta'(958)}{f_2(1910) \rightarrow \pi\pi}$	0.01
$\frac{f_2(1910) \rightarrow K\bar{K}}{f_2(1910) \rightarrow \pi\pi}$	0.31

# $f_2(1950)$

- The meson  $f_2(1950)$  decays into  $\eta\eta$ ,  $\pi\pi$ ,  $K^*K^*$  and  $K\bar{K}$  pairs, and the experimental ratio  $\eta\eta/\pi\pi$  of  $0.14 \pm 0.05$  agrees with theory.
- Nevertheless, its huge total decay width of 460 MeV seems at odds for a tensor glueball candidate.

Branching Ratio	eLSM
$\frac{f_2(1950) \rightarrow \bar{K}^*(892) K^*(892)}{f_2(1950) \rightarrow \pi\pi}$	42
$\frac{f_2(1950) \rightarrow \eta\eta}{f_2(1950) \rightarrow \pi\pi}$	0.081
$\frac{f_2(1950) \rightarrow \bar{K}K}{f_2(1950) \rightarrow \pi\pi}$	0.32

- the resonance  $f_2(2010)$  has a total decay width of  $202 \pm 60$  MeV.
- Yet, only  $K\bar{K}$  and  $\phi(1020)\phi(1020)$  decays have been seen.
- This suggests a large strange-antistrange content for this resonance, rather than a predominantly gluonic state.



- In view of the LQCD prediction for the tensor glueball mass around 2.2 GeV, one of the closest resonances is  $f_2(2150)$ .
- However, the ratio  $K\bar{K}/\eta\eta$  is  $1.28 \pm 0.23$ , while the theoretical prediction is about 4.
- Similarly, the ratio of  $\pi\pi/\eta\eta$  is experimentally less than 0.33, while the theoretical estimate is about 10.

Branching Ratio	eLSM
$\frac{f_2(2150) \rightarrow \eta\eta}{f_2(2150) \rightarrow \pi\pi}$	0.1
$\frac{f_2(2150) \rightarrow K\bar{K}}{f_2(2150) \rightarrow \pi\pi}$	0.38

- The meson  $f_J(2220)$  (with  $J = 2$  or  $4$ ) is historically treated as a good candidate. However, most of the decays that the theory predicts are not seen experimentally.
- PDG data offers one branching ratio for  $\pi\pi/K\bar{K}$  of  $1.0 \pm 0.5$ , while the theoretical prediction is about 2.5.
- For these reasons we conclude it should not be considered as a good candidate any longer.

# $f_2(2300)$

- The resonance  $f_2(2300)$ , with a total width of  $149 \pm 41$  MeV, decays only into  $K\bar{K}$  and  $\phi\phi$ , thus suggesting that it is predominantly a strange-antistrange object.
- We have a prediction for the branching ratio, but no data to compare with

Branching Ratio	eLSM
$\frac{f_2(2300) \rightarrow \bar{K} K}{f_2(2300) \rightarrow \phi(1020), \phi(1020)}$	0.06

# $f_2(2340)$

- Initially, the resonance  $f_2(2340)$  decays into  $\eta\eta$  and  $\phi(1020)\phi(1020)$  that may also imply a large strange-antistrange component
- Moreover, it has a quite large total decay width of  $322 \pm 60$  MeV, which does not favor its interpretation as a gluonic state.

Branching Ratio	eLSM
$\frac{f_2(2340) \rightarrow \eta\eta}{f_2(2340) \rightarrow \phi(1020)\phi(1020)}$	0.02

# Summary

- Glueballs are a yet undiscovered prediction of QCD and an active research topic of both theoretical models and experimental efforts
- We have adapted the eLSM for tensor mesons to describe the tensor glueball
- We obtain branching ratios; vector channels are dominant, in particular  $\rho\rho$  and  $K^*K^*$
- However, none of the resonances looked at fit the picture painted by the model
- Sometimes data is limited, in particular, the analysis for the states  $f_J(2220)$ ,  $f_2(2300)$ , and  $f_2(2340)$  would benefit from more experimental data.

**Thank you for your attention**