Complexity, high-energy Collisions, and differential observables

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UJK Uniwersytet

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High-energy Physics

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- In this lecture we discuss the nonadditive entropy proposed by Constantino Tsallis in 1988.

C. Tsallis, J. Stat. Phys. 52, 479 (1988)

Entropy

Boltzmann-Gibbs

$$S_{\mathrm{BG}} = -\sum_i p_i \ln p_i$$

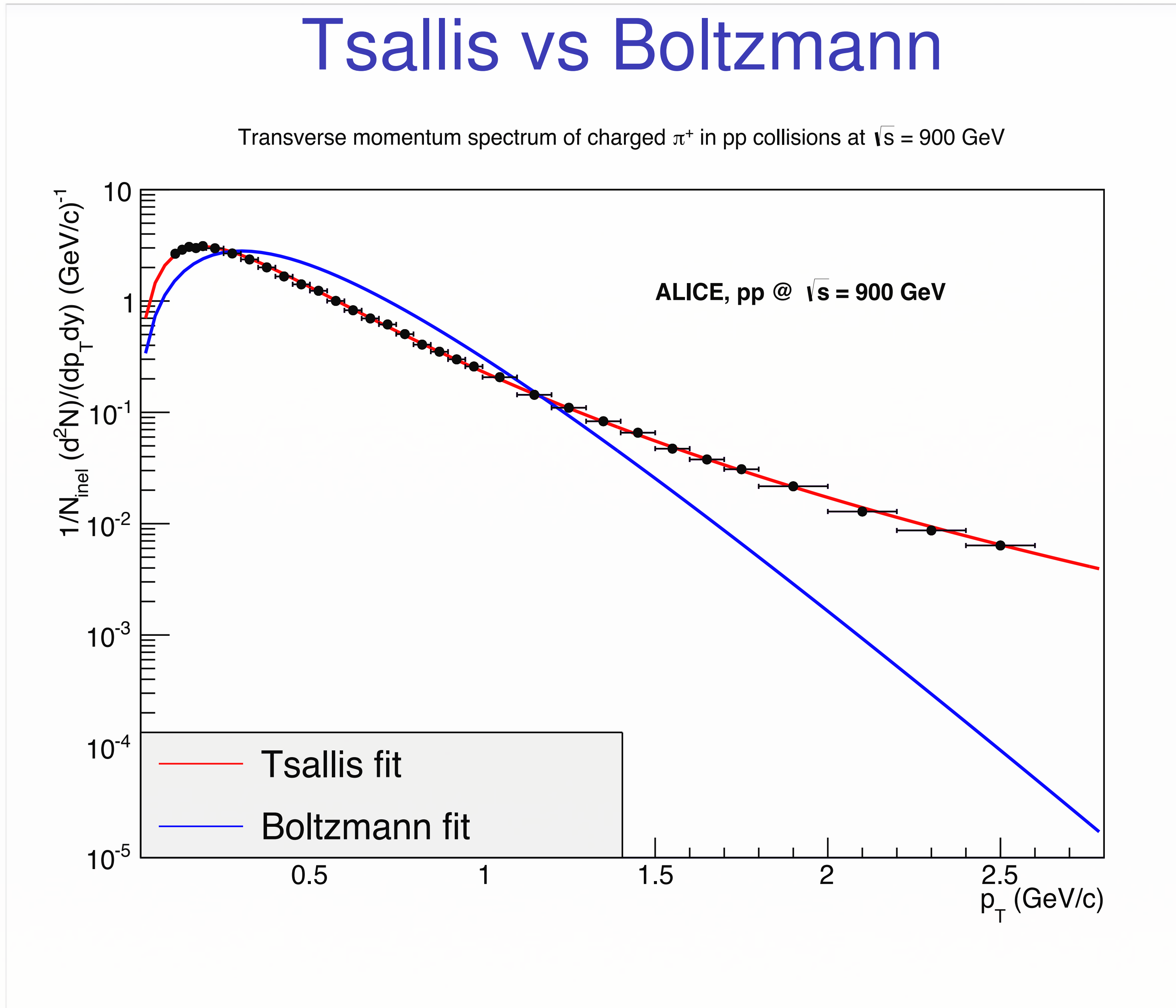
$q \to 1$

Tsallis

$$S_{\mathrm{T}} = -\sum_i p_i^q \ln_q p_i$$

$$\ln_q p_i = \frac{1 - p_i^{1-q}}{q - 1} \quad q \in \mathbb{R}^{\geq}$$

Complexity in high-energy collisions



Complexity in high-energy collisions

communications physics

A Nature Portfolio journal

Article

https://doi.org/10.1038/s42005-025-01973-x

Lévy walk of pions in heavy-ion collisions

Check for updates

Dániel Kincses, Márton Nagy & Máté Csanád

Complexity in high-energy collisions

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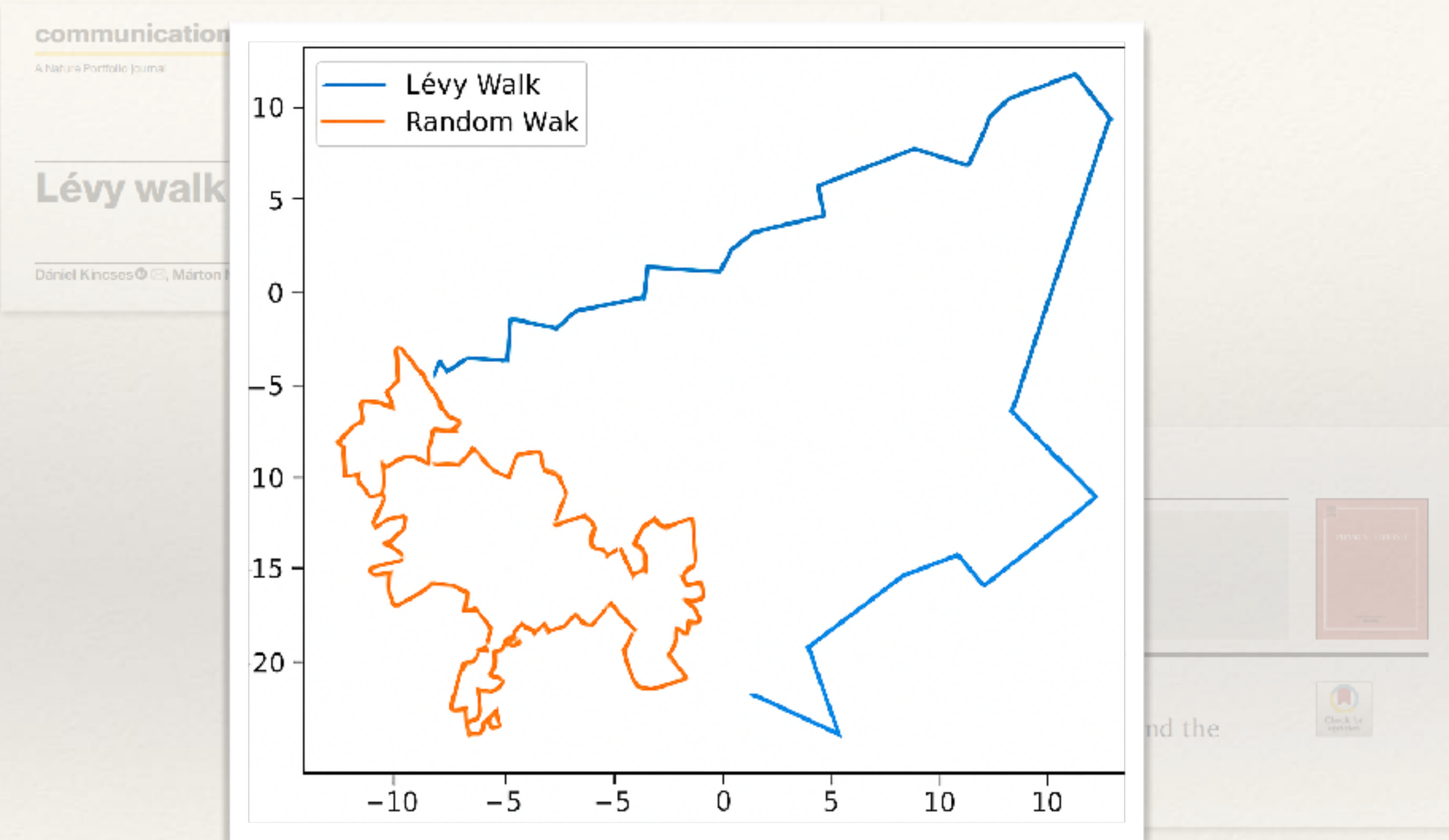
Letter

Jet quenching of the heavy quarks in the quark-gluon plasma and the nonadditive statistics

Trambak Bhattacharyya [a,b,*], Eugenio Megías [c], Airton Deppman [d]

Complexity in high-energy collisions



Summary I

- Systems created in high-energy collisions are complex

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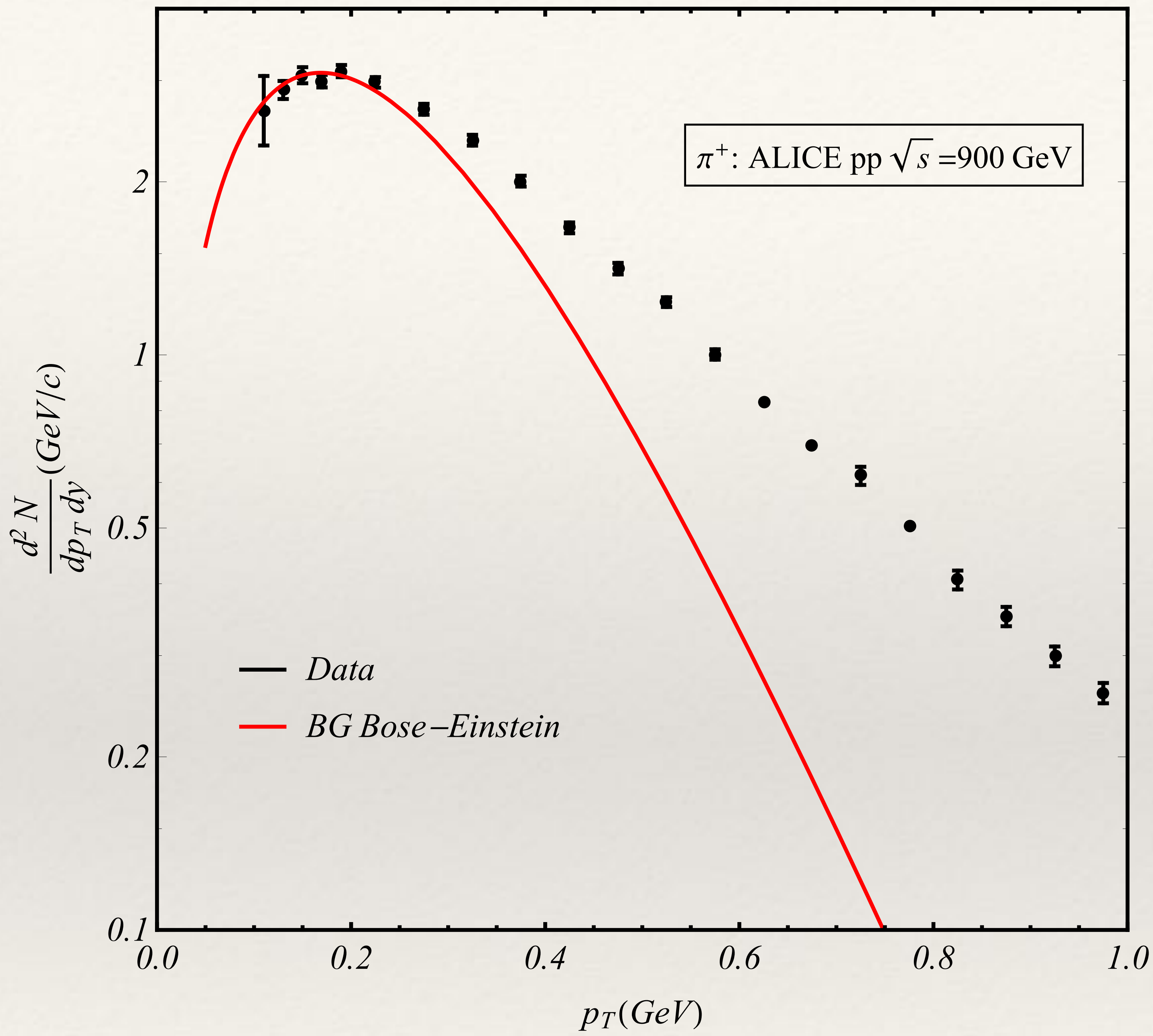
- In complex systems, extreme events are not so ‘rare’

Complexity and observables



An observable and theoretical modelling

$$\frac{d^2N}{dp_{\mathrm{T}}dy} = \frac{gV}{(2\pi)^3}\int_0^{2\pi} d\varphi \; p_{\mathrm{T}} \; \epsilon_{\mathbf{p}} \; \langle n_{\mathbf{p}} \rangle$$

An observable and theoretical modelling

Single-particle distribution

$$\frac{d^2N}{dp_{\mathrm{T}}dy} = \frac{gV}{(2\pi)^3}\int_0^{2\pi} d\varphi \; p_{\mathrm{T}} \; \epsilon_{\mathbf{p}} \; \langle n_{\mathbf{p}} \rangle$$

We observe that the Boltzmann-Gibbs Bose-Einstein single-particle distribution does not describe pion data

$$\left.\frac{d^2N}{dp_T\,dy}\right|_{y=0,\mu=0} = gV\frac{p_\mathrm{T}m_\mathrm{T}}{(2\pi)^2}\frac{1}{e^{\frac{m_\mathrm{T}}{T}}-1}$$

Is there any generalized Bose-Einstein single-particle distribution that explains pion data?

SPDs obtained from maximization

C. Tsallis, J. Stat. Phys. 52, 479 (1988)

Tsallis entropy

$$S_{\mathrm{T}} = -\sum_i p_i^q \ln_q p_i$$

$$\ln_q p_i = \frac{1 - p_i^{1-q}}{q - 1} \quad q \in \mathbb{R}^{\geq}$$

- Normalization of probabilities (constraint)

$$\varphi = \sum_i p_i - 1 = 0$$

- Define averaging scheme *Tsallis et al. proposed three schemes (C. Tsallis, R. Mendes, A.R. Plastino, Physics A 261, 534 (2018)).*

$$\langle A \rangle = \sum_i p_i A_i$$

- Define potential $\Omega = \langle H \rangle - TS - \mu \langle N \rangle$

- Extremise $\Phi = \Omega - \lambda\phi$ w. r. t. p_i to solve for

$$\varphi = \sum_i p_i - 1 = 0$$

$$p_i = \left(1 + \frac{q-1}{q}\frac{\Lambda - E_i + \mu N_i}{T}\right)^{\frac{1}{q-1}}$$

$$\Lambda = \lambda - T$$

Equilibrium set of probabilities

$$p_i = \frac{1}{\Gamma\left(\frac{1}{1-q}\right)}\int_0^\infty t^{\frac{q}{1-q}} e^{-t\left(1 + \frac{q-1}{q}\frac{\Lambda - E_i + \mu N_i}{T}\right)}\, dt$$

Integral representation

$$\sum_i p_i = \frac{1}{\Gamma\left(\frac{1}{1-q}\right)} \int_0^\infty t^{\frac{q}{1-q}} e^{-t\left(1 + \frac{q-1}{q} \frac{\Lambda - \Omega_{\mathrm{G}}(\beta')}{T}\right)} dt = 1$$

$$\beta' = t(1-q)/qT, \quad \Omega_{\mathrm{G}}(\beta') = -\frac{1}{\beta'} \ln Z_{\mathrm{G}}(\beta'),$$

$$\text{and} \quad Z_{\mathrm{G}}(\beta') = \sum_i e^{-\beta'(E_i - \mu N_i)}$$

Probability normalization

Nonadditive SPD expressed in terms of BG SPD

$$\langle n_{\mathbf{p}} \rangle = \frac{1}{\Gamma\left(\frac{q}{q-1}\right)} \int_0^\infty t^{\frac{q}{q-1}} \, e^{-t} \, e^{\beta' \Lambda} \, Z_{\mathrm{G}}(\beta') \, \langle n_{\mathbf{p}} \rangle_{\mathrm{G}} \, dt$$

for $q < 1$

Single-mode simple harmonic oscillator

$$\langle n_{\mathbf{p}} \rangle = \frac{1}{\Gamma\left(\frac{q}{q-1}\right)} \int_0^\infty t^{\frac{q}{q-1}} \, e^{-t} \, e^{\beta' \Lambda} \, Z_{\mathrm{G}}(\beta') \, \langle n_{\mathbf{p}} \rangle_{\mathrm{G}} \, dt$$

for $q < 1$

$$Z_{\mathrm{G}}(\beta) = \frac{1}{1 - \exp(-\beta \epsilon_{\mathbf{p}})}$$

$$\langle n_{\mathbf{p}} \rangle_{\mathrm{G}} = \frac{1}{\exp(\beta \epsilon_{\mathbf{p}}) - 1}$$

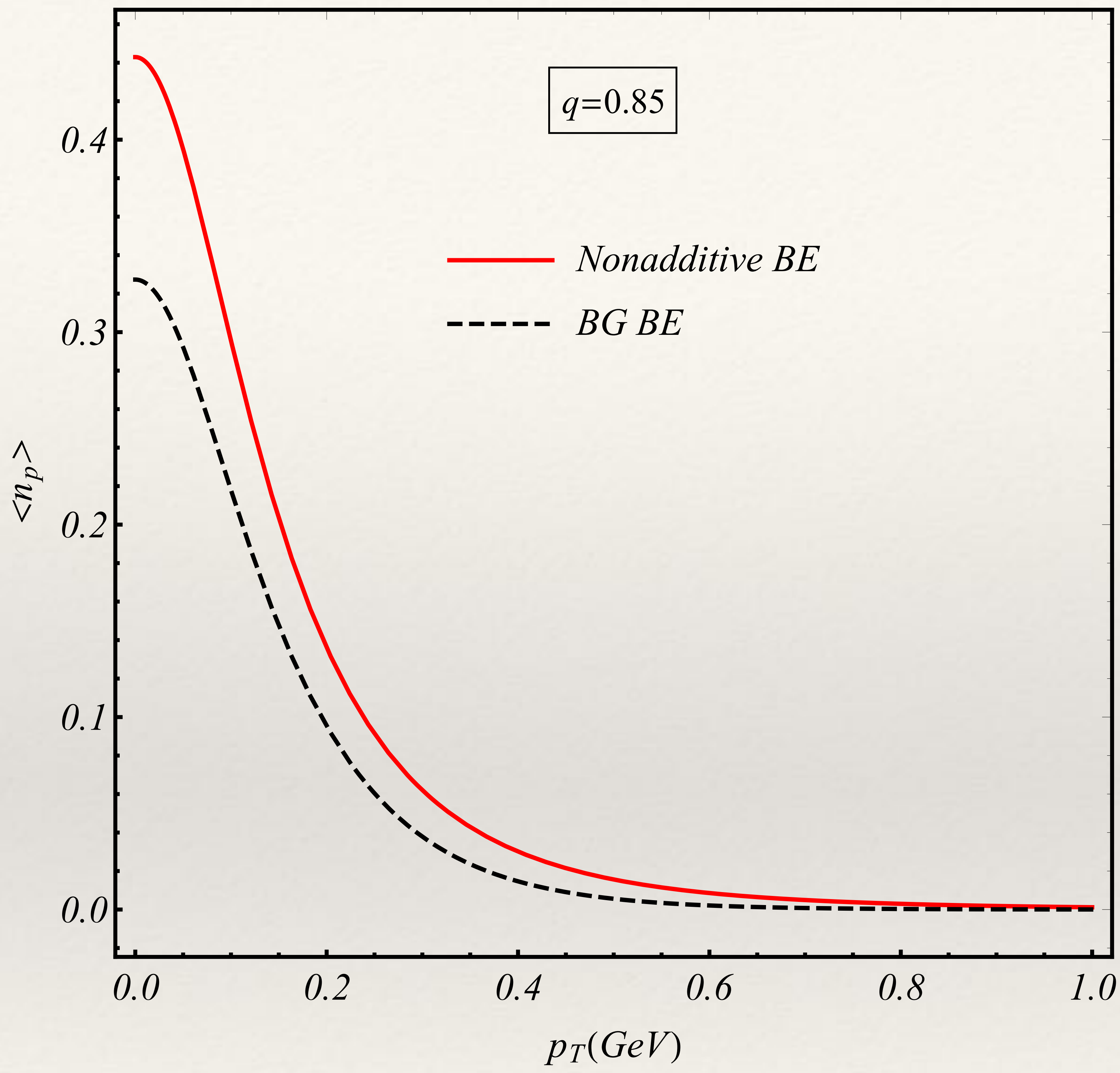
Single-mode simple harmonic oscillator

$$\langle n_{\mathbf{p}} \rangle = \left(\frac{qT}{(1-q)\epsilon_{\mathbf{p}}} \right)^{\frac{1}{1-q}} \zeta\left(\frac{q}{1-q}, \frac{qT}{(1-q)\epsilon_{\mathbf{p}}} - \frac{\Lambda}{\epsilon_{\mathbf{p}}} \right)$$

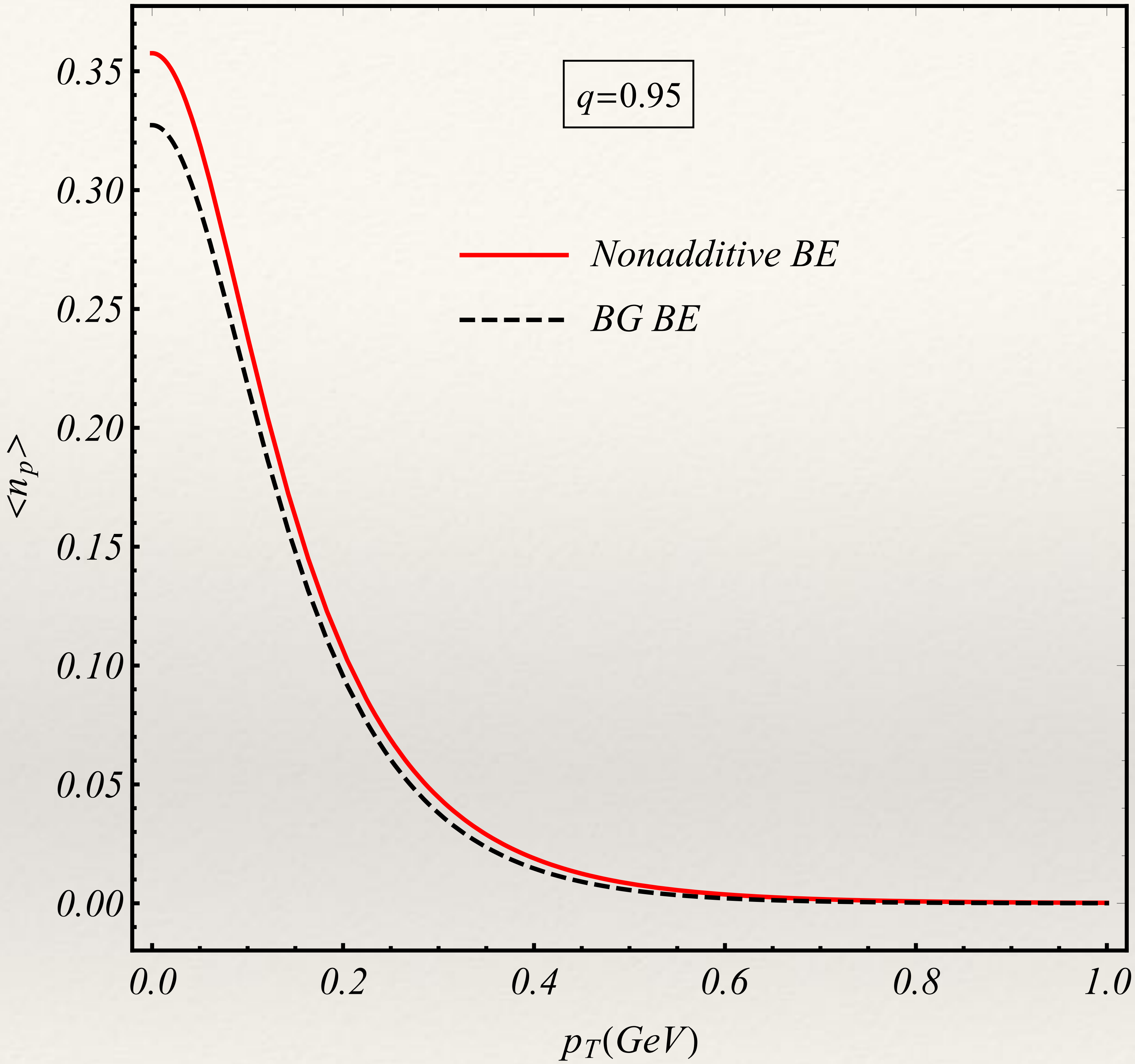
$$- \left(\frac{qT}{(1-q)\epsilon_{\mathbf{p}}} - \frac{\Lambda}{\epsilon_{\mathbf{p}}} \right) \left(\frac{qT}{(1-q)\epsilon_{\mathbf{p}}} \right)^{\frac{1}{1-q}} \zeta\left(\frac{1}{1-q}, \frac{qT}{(1-q)\epsilon_{\mathbf{p}}} - \frac{\Lambda}{\epsilon_{\mathbf{p}}} \right)$$

The value of Λ is obtained from probability normalization

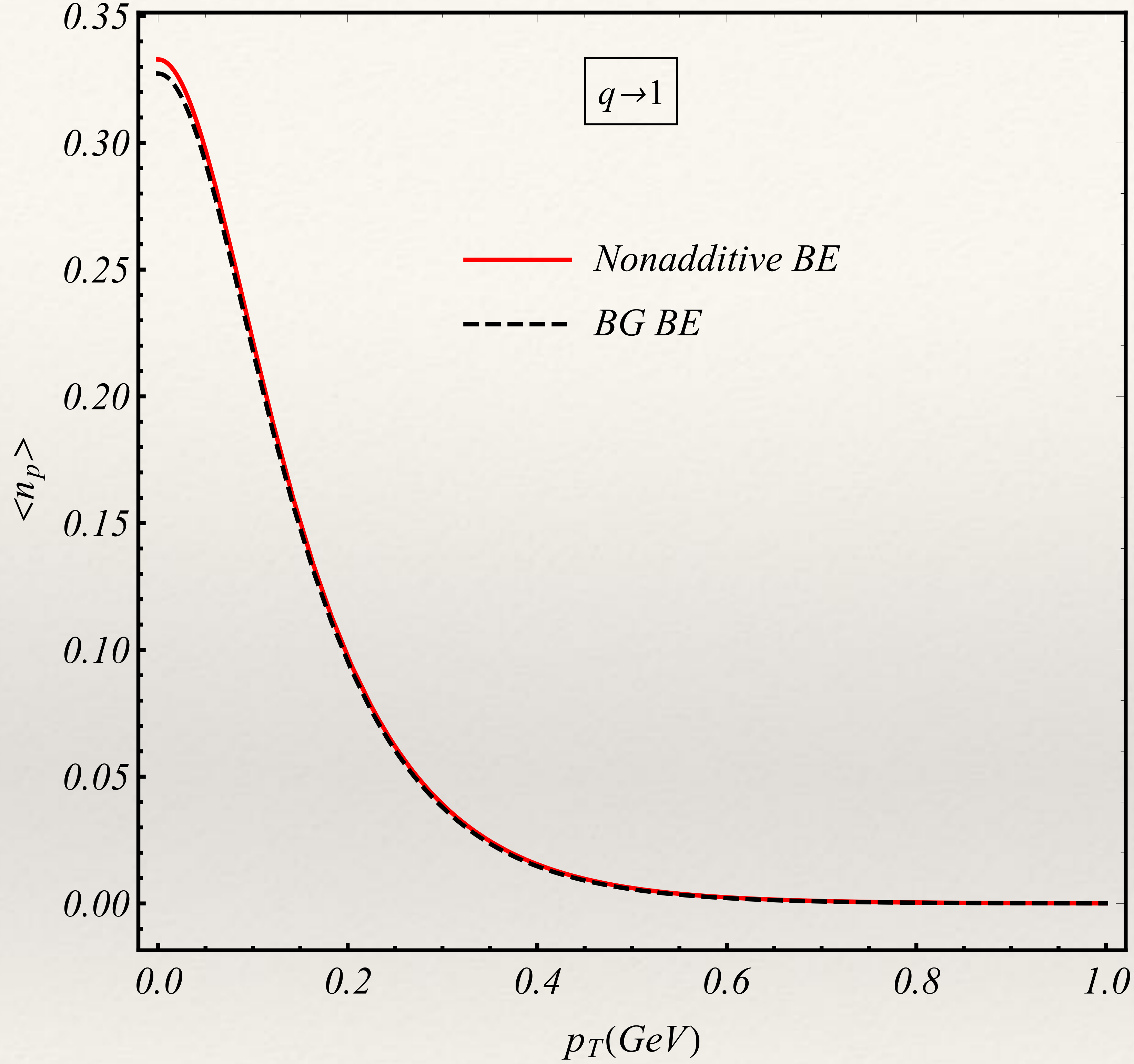
So, we have derived a generalized Bose-Einstein distribution



q=0.85
Nonadditive BE
BG BE
0.4
0.3
0.2
0.1
0.0
<n_p>
0.0
0.2
0.4
0.6
0.8
1.0
p_T(GeV)

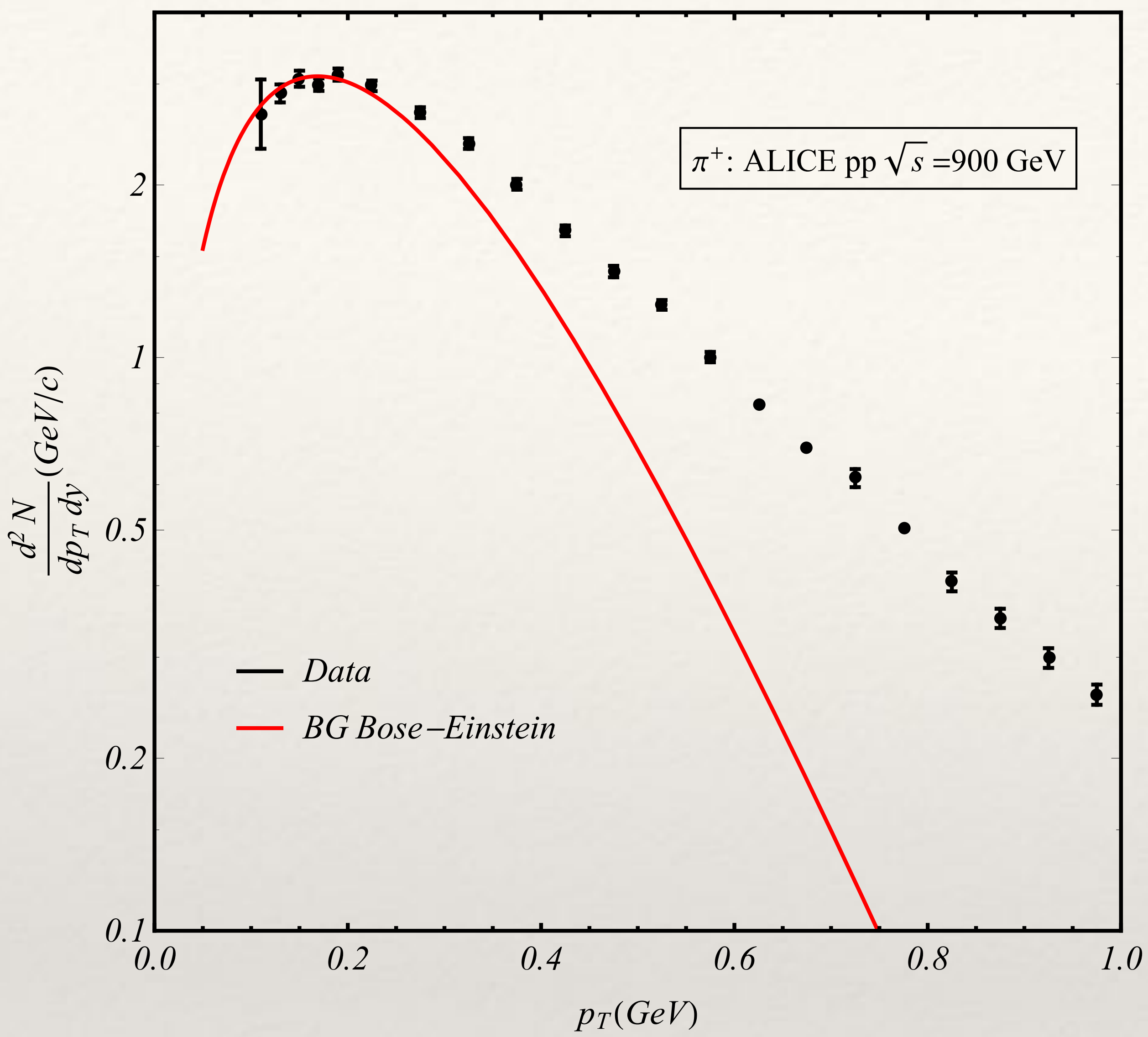


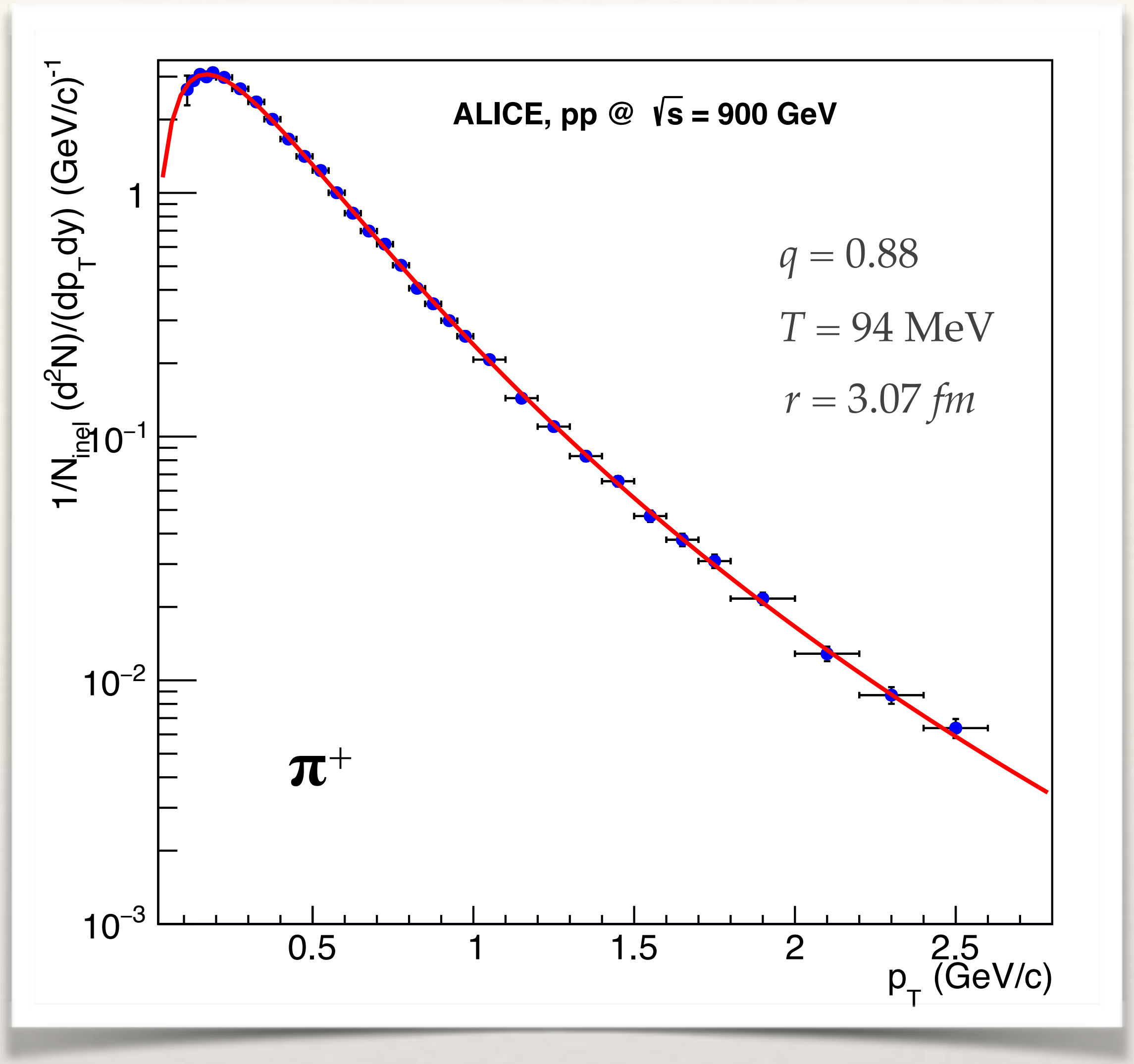
q=0.95
Nonadditive BE
BG BE
<n_p>
p_T(GeV)
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0
0.2
0.4
0.6
0.8
1.0


q→1
Nonadditive BE
BG BE
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
<n_p>
0.0
0.2
0.4
0.6
0.8
1.0
p_T(GeV)

The generalized Bose-Einstein distribution is clearly different from the Boltzmann-Gibbs Bose-Einstein distribution when $q \neq 1$

Does it describe pion data?





T. Bhattacharyya, M. Rybczyński, G. Wilk, and Z. Włodarczyk, Phys. Lett. B ***867*** *, 139588 (2025)*

Summary II and a brief outlook

- Systems created in high-energy collisions are complex
- Manifestations through power-law distributions
- While describing such a system, we derive, from a generalised entropy, a power-law Bosonic distribution describing pion data that the conventional exponential Bosonic distribution fails to describe
- This generalised BE distribution is not phenomenological
- Also, compare with the superstatistics approach

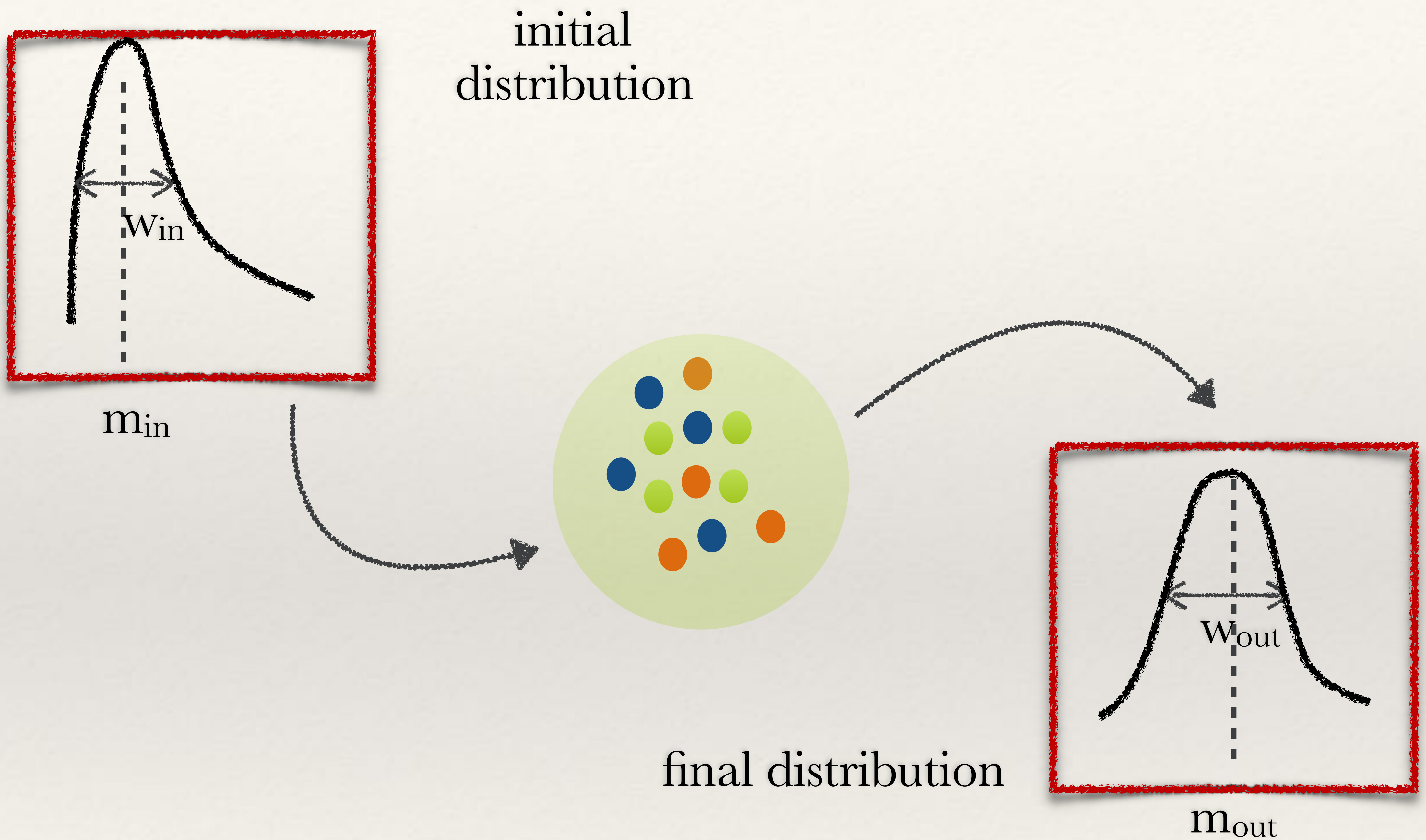
G. Wilk, Z. Włodarczyk, Phys. Rev. Lett. ***84****, 2770(2000)* *C. Beck, E. G. D. Cohen, Physica A* ***322****, 267 (2003)*

More observables: I

$$R_{AA} = \frac{\left(d^2N/dp_T dy\right)^{A+A}}{N_{coll} \times \left(d^2N/dp_T dy\right)^{p+p}}.$$

Nuclear suppression factor

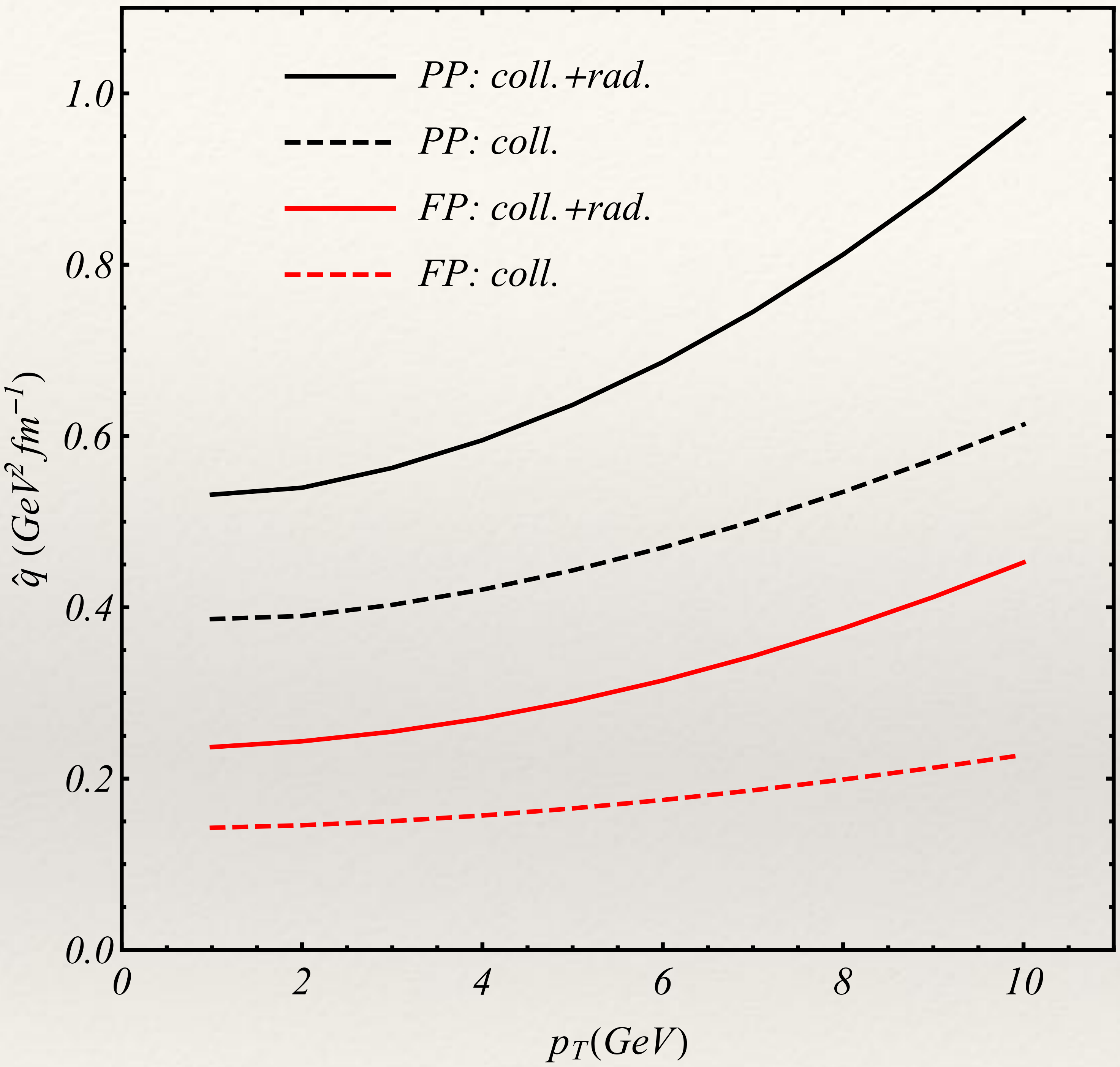
Theoretical modelling



$$R^{\mathrm{h}}_{\mathrm{AA}}(P_{\mathrm{T}},\ell) \simeq \exp\left[-\frac{2\alpha_s C_F}{\sqrt{\pi}}\ell\sqrt{\frac{\hat{q}\mathcal{L}^{\mathrm{abs}}_h}{P_{\mathrm{T}}}} + \frac{16\alpha_s C_F}{9\sqrt{3}}\ell\left(\frac{\hat{q}m^2}{m^2+P_{\mathrm{T}}^2}\right)^{1/3}\right].$$

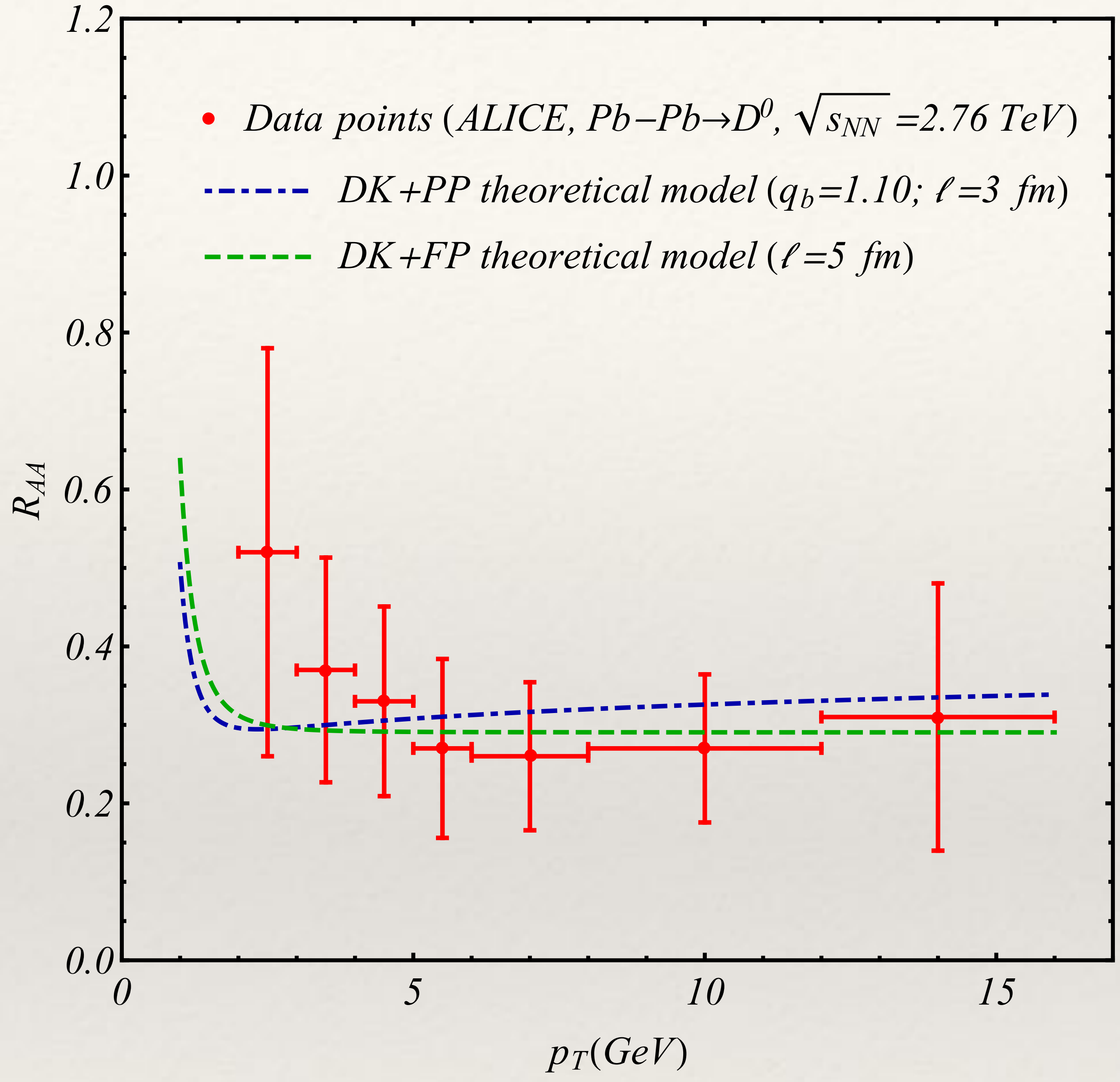
Y. Dokshitzer and D. Kharzeev, Phys. Lett. B ***519*** *, 199 (2001)*

Jet quenching parameter



$$\hat{q} = \frac{\langle\langle k_T^2 \rangle\rangle}{v_L} \sim B_\perp$$

T. Bhattacharyya, E. Megías, and A. Deppman, Phys. Lett. B ***856*** *, 138907 (2024)*



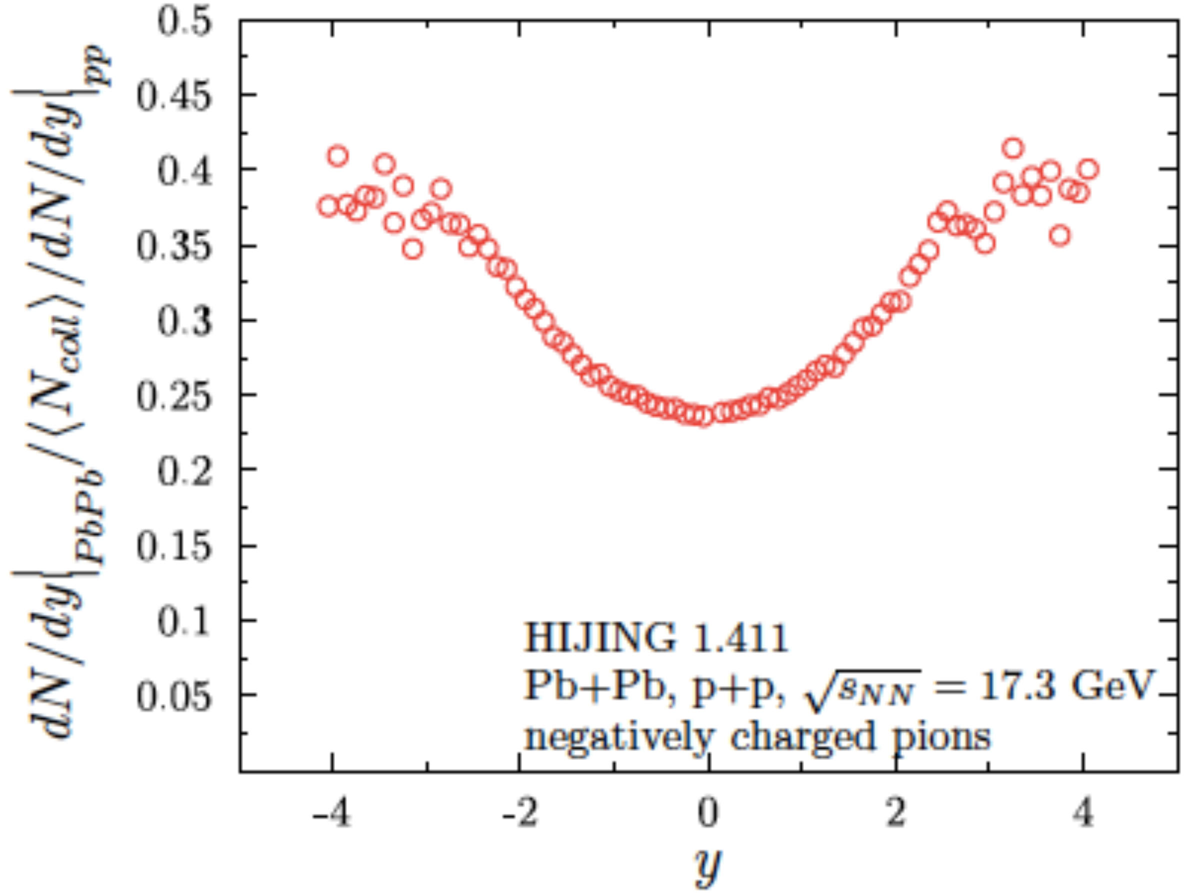
T. Bhattacharyya, E. Megías, and A. Deppman, Phys. Lett. B **856** , 138907 (2024)

More observables: II

$$R_{dN/dy} = \frac{\frac{dN^{\text{Pb-Pb}}}{dy}}{\langle N_{\text{coll}} \rangle \frac{dN^{pp}}{dy}}$$

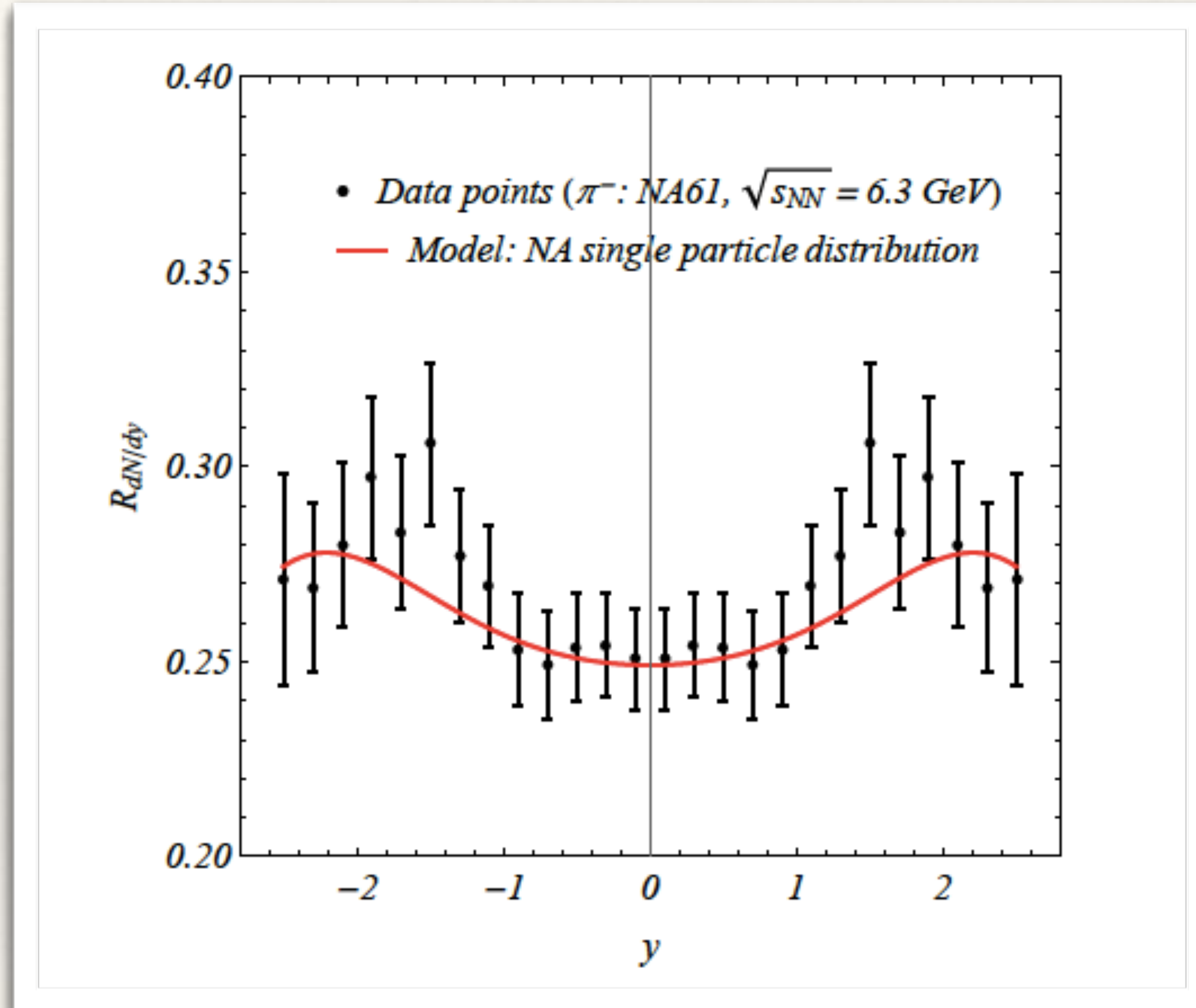
Longitudinal suppression factor

Approach I: numerical modelling



T. Bhattacharyya, M. Rybczyński, and Z. Włodarczyk, arXiv: 2504.02548 [nucl-th]

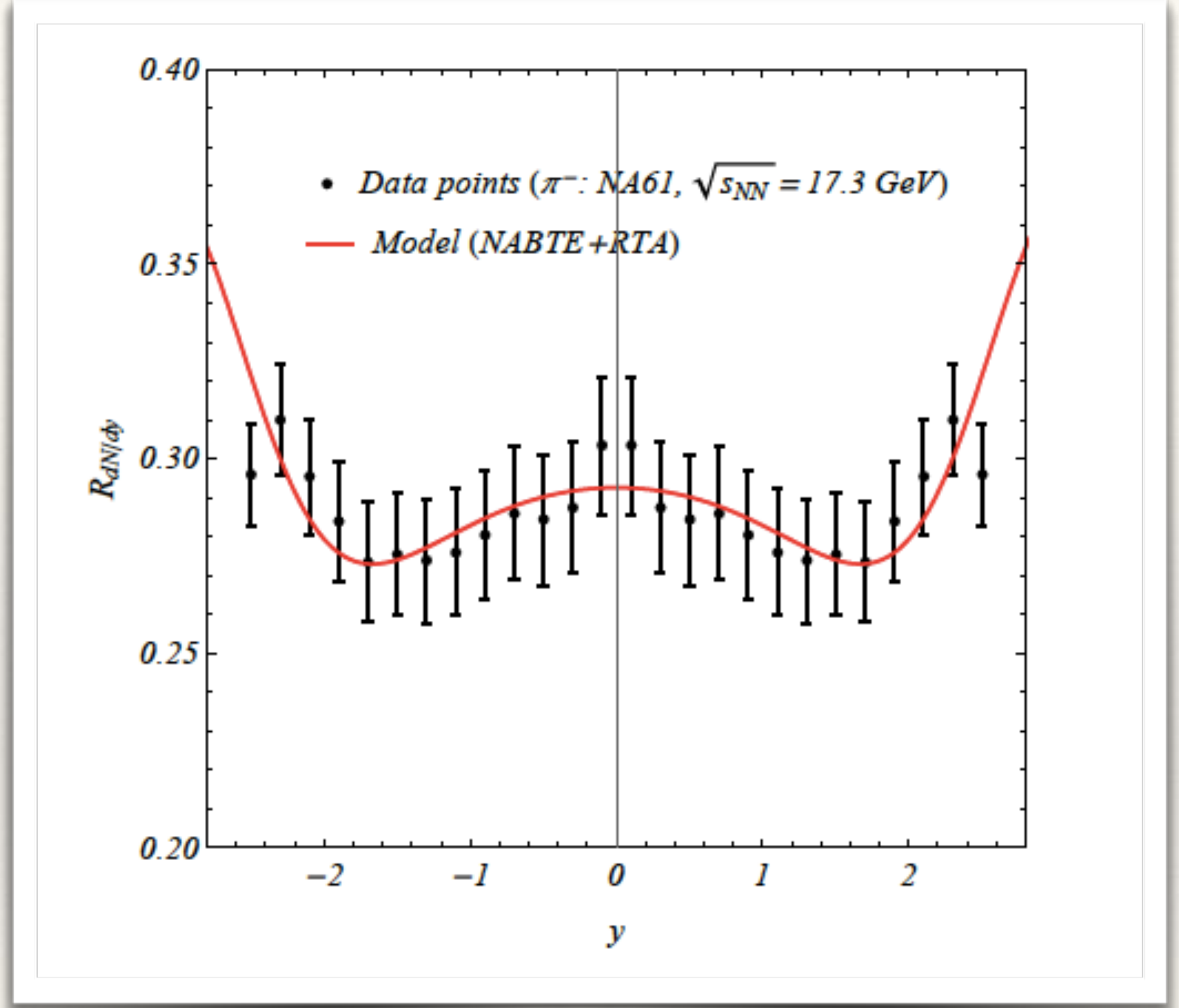
Approach 2: phenomenological



q_1=1.12,
q_2=1.10,
T_1=0.086 GeV,
T_2=0.101 GeV,
V_1=75.9 GeV^{-3},
V_2= 10^4 GeV^{-3},
N_{coll}=808

T. Bhattacharyya, M. Rybczyński, and Z. Włodarczyk, arXiv: 2504.02548 [nucl-th]

Approach 3: transport



q_{in}=1.06,
q=1.01,
T_{in}=0.08 GeV,
T=0.09 GeV,
t/τ=0.839,
μ_{in}= 0.16 GeV,
μ= 0.04 GeV

T. Bhattacharyya, M. Rybczyński, and Z. Włodarczyk, arXiv: 2504.02548 [nucl-th]

Final summary and outlook

- Complexity in systems created in high-energy collisions is manifested through power-law differential observables
- We discussed three examples: a) transverse momentum spectra; b) transverse nuclear suppression; c) longitudinal nuclear suppression
- Starting from a nonadditive entropy, we derived a generalized Bose-Einstein distribution suitable to describe bosonic spectra produced in high-energy collisions.
- We also outlined a modified transport approach for describing nuclear suppression
- More rigorous approaches: numerical solutions with the input of transport coefficients, collectivity, centrality dependence of path length, comparison with interferometry, connection with quantum field theory

Acknowledgements

A. Deppman, E. Megías, M. Rybczyński, G. Wilk, Z. Włodarczyk

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Thank you !!