Scalars with a static classical source are not trivial after all

Outline

- Long introduction: Why is it surprising in the first place?
- Exact solution and practical application: Schwinger-like vacuum decay, production of exotic particles
- Solving the paradox: what was missing in the historical arguments for trivial dynamics

Leonardo Tinti, Kielce, 5/6/'24

LT, A Vereijken, S Jafarzade, F Giacosa, arxiv:2403.15531

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Another case of on-trivial vacuum behavior

A cautionary tale regarding popular assumptions, and taking them for granted

From a discussion between Sinyukov and Akkelin regarding the statistical non-equilibrium density operator

$$\hat{\rho}_{NEDO} = \frac{e^{-\hat{\Upsilon}}}{Z_{\Upsilon}} = \frac{1}{Z_{\Upsilon}} \exp\left\{ -\int d\Sigma_{\mu} \frac{\hat{T}^{\mu\nu} u_{\nu}}{T} \right\}$$

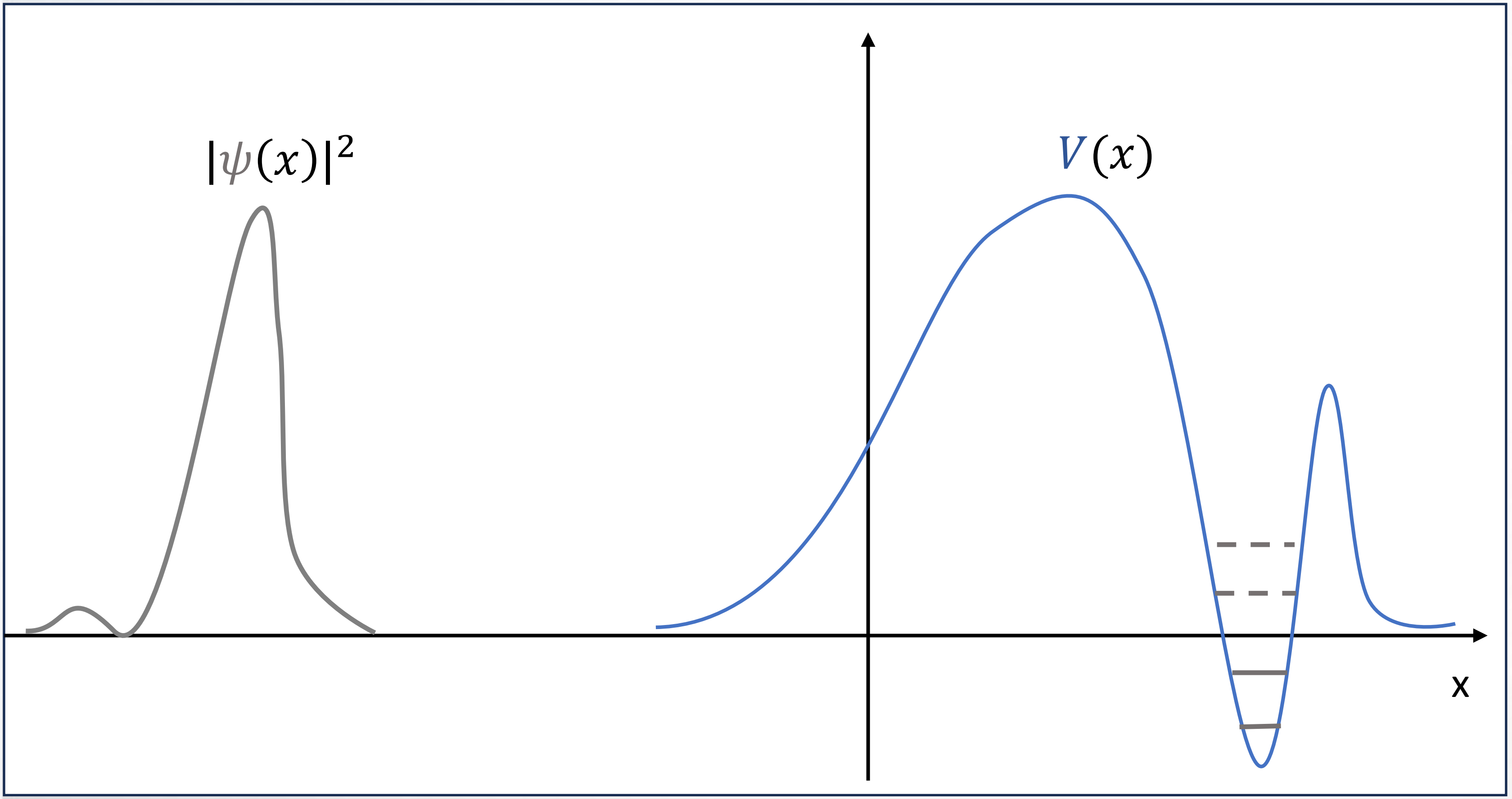
The proper way to renormalize the operators to get the expectation values starts from removing the minimum of the $\hat{\Upsilon}$ operator, not the Hamiltonian (Minkowski vacuum)

But the particle number operator (and momentum etc.) is simply normal ordered:

The ``Υ vacuum '' contributes to the total number of particles. (similar to the Unrhu vacuum contribution)

S V Akkelin, 10.1140/epja/i2019-12755-9, 10.1103/PhysRevD.103.116014

How the heck are relativistic scalar fields supposed not to see a static classical source? (non-relativistic particles see them!)



Staring from the S-matrix idea

$$\hat{H} = \frac{\hat{P}^2}{2\,M} + V(\hat{X}) \neq \frac{\hat{P}^2}{2M} = \hat{H}_0$$

Transition probability amplitudes between asymptotically-free states $\psi_{in/out}$

$$\lim_{t\to\pm\infty} \psi_{in\backslash\text{out}} = \psi_{free}$$

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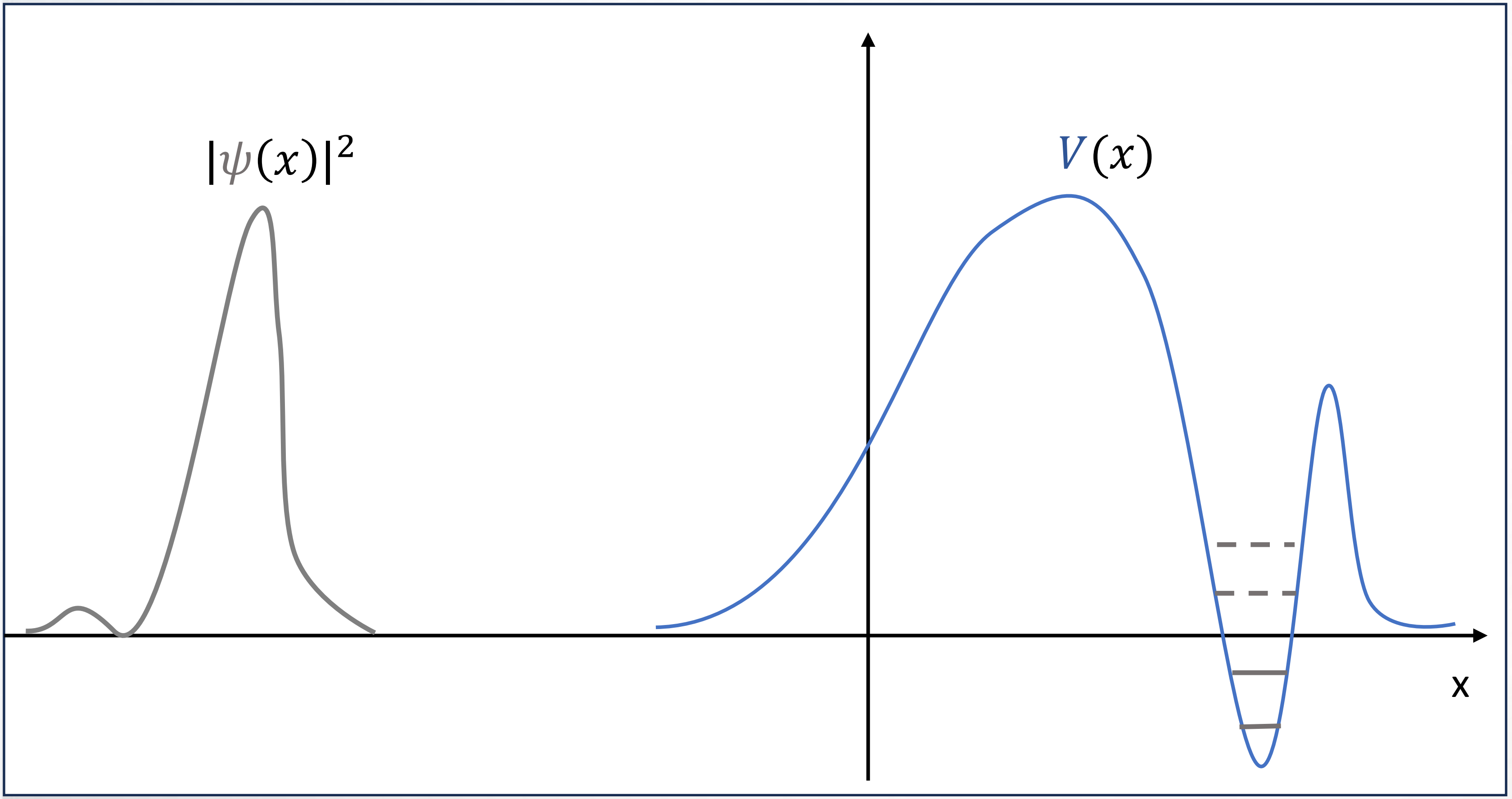
In quantum mechanics it is only relevant the action of the operators over the wavefunctions $e^{-\hat{H}(t-t_0)}\psi(t_0,x)$

Staring from the S-matrix idea



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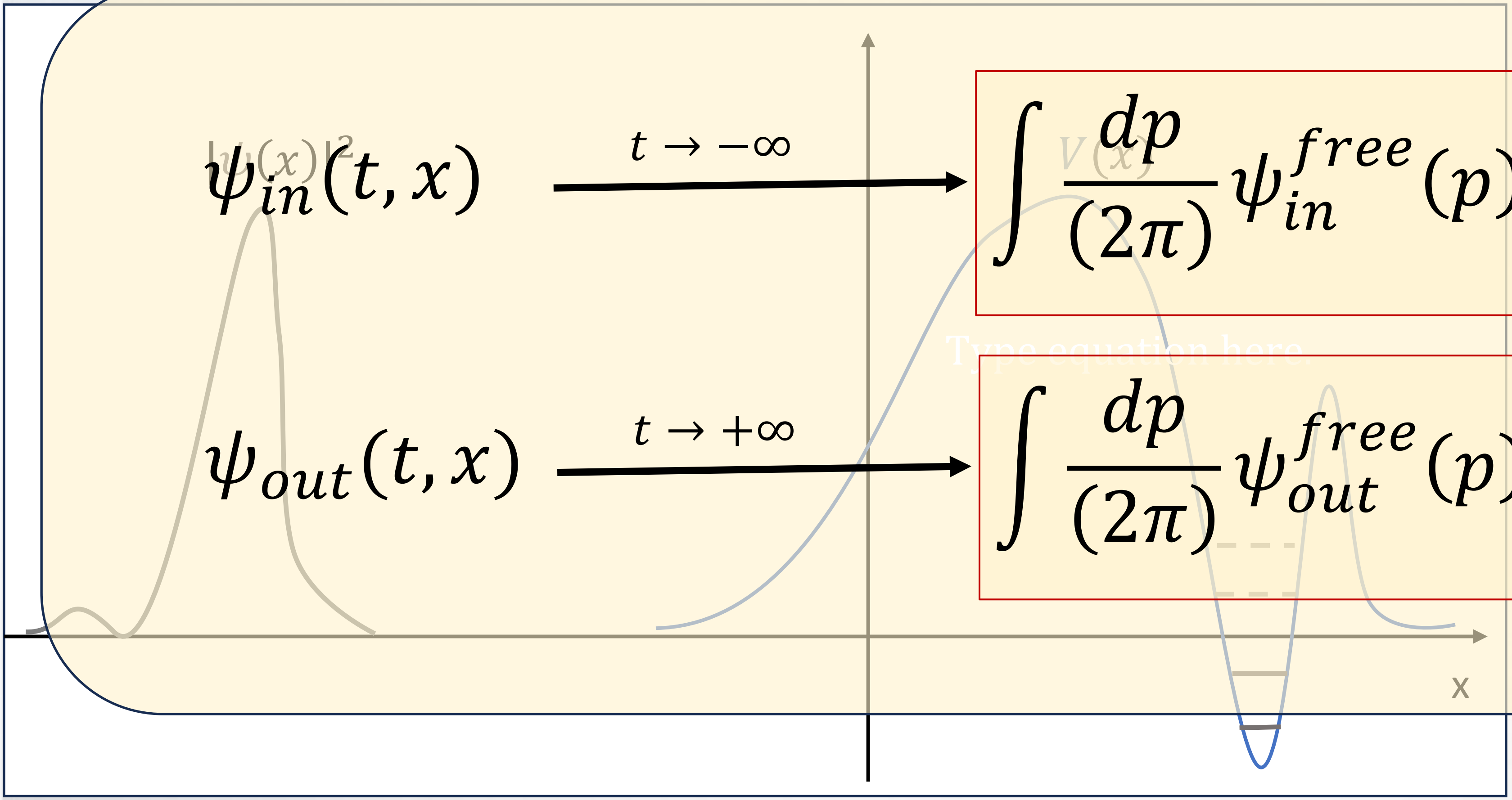

$$\psi_{in}(t,x) \xrightarrow{t\to-\infty} \int \frac{dp}{(2\pi)}\psi_{in}^{free}(p)\, e^{-iE_p t+ipx}$$

$$\psi_{out}(t,x) \xrightarrow{t\to+\infty} \int \frac{dp}{(2\pi)}\psi_{out}^{free}(p)\, e^{-iE_p t+ipx}$$

$$\hat{H} = \frac{\hat{P}^2}{2M} + V(\hat{X}) \neq \frac{\hat{P}^2}{2M} = \hat{H}_0$$

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$$\lim_{t\to\pm\infty}\psi_{in\backslash out} = \psi_{free}$$

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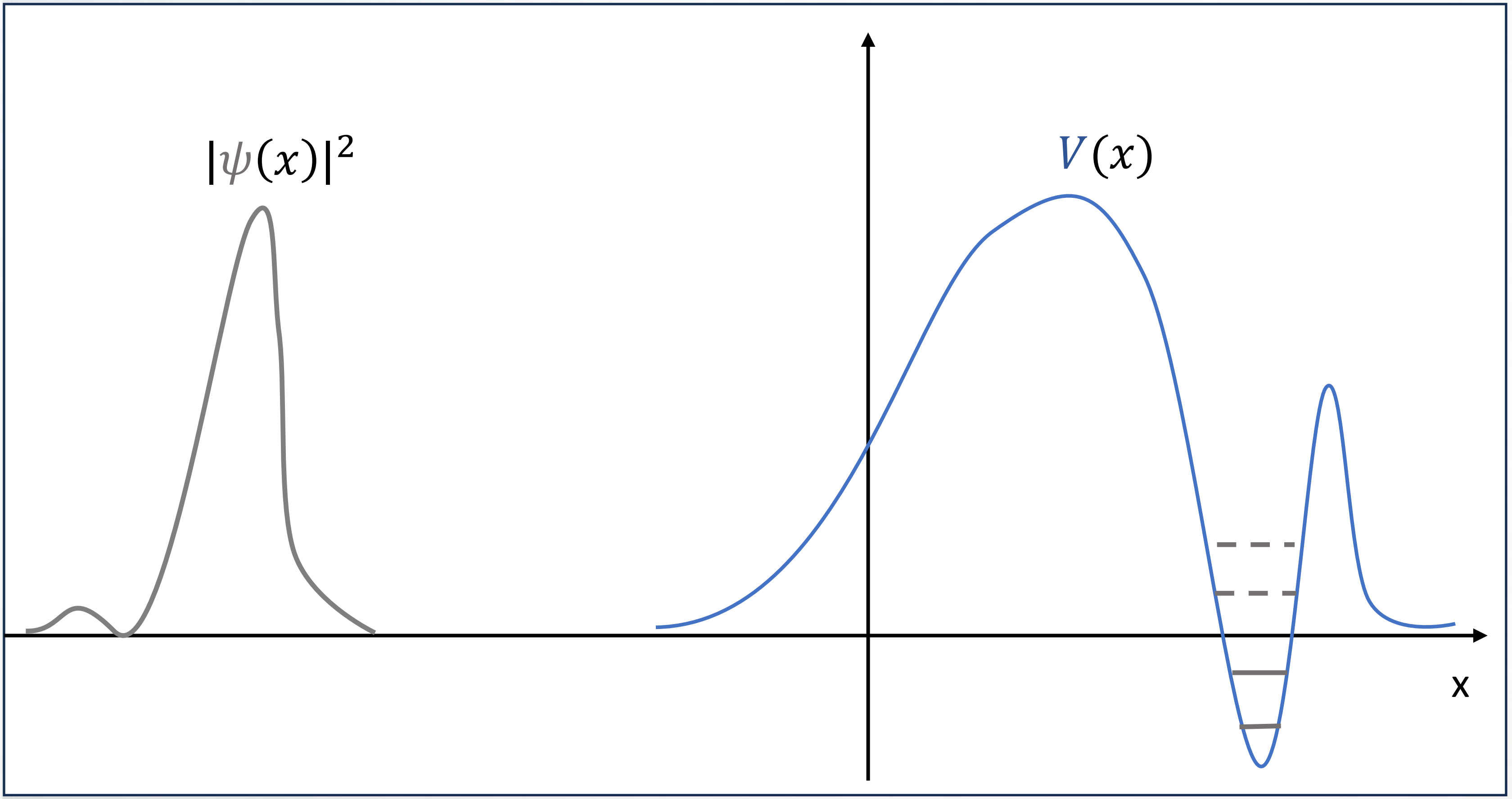


$$\langle\psi_{out}|\psi_{in}\rangle = \langle\psi_{out}^{free}|S|\psi_{in}^{free}\rangle$$

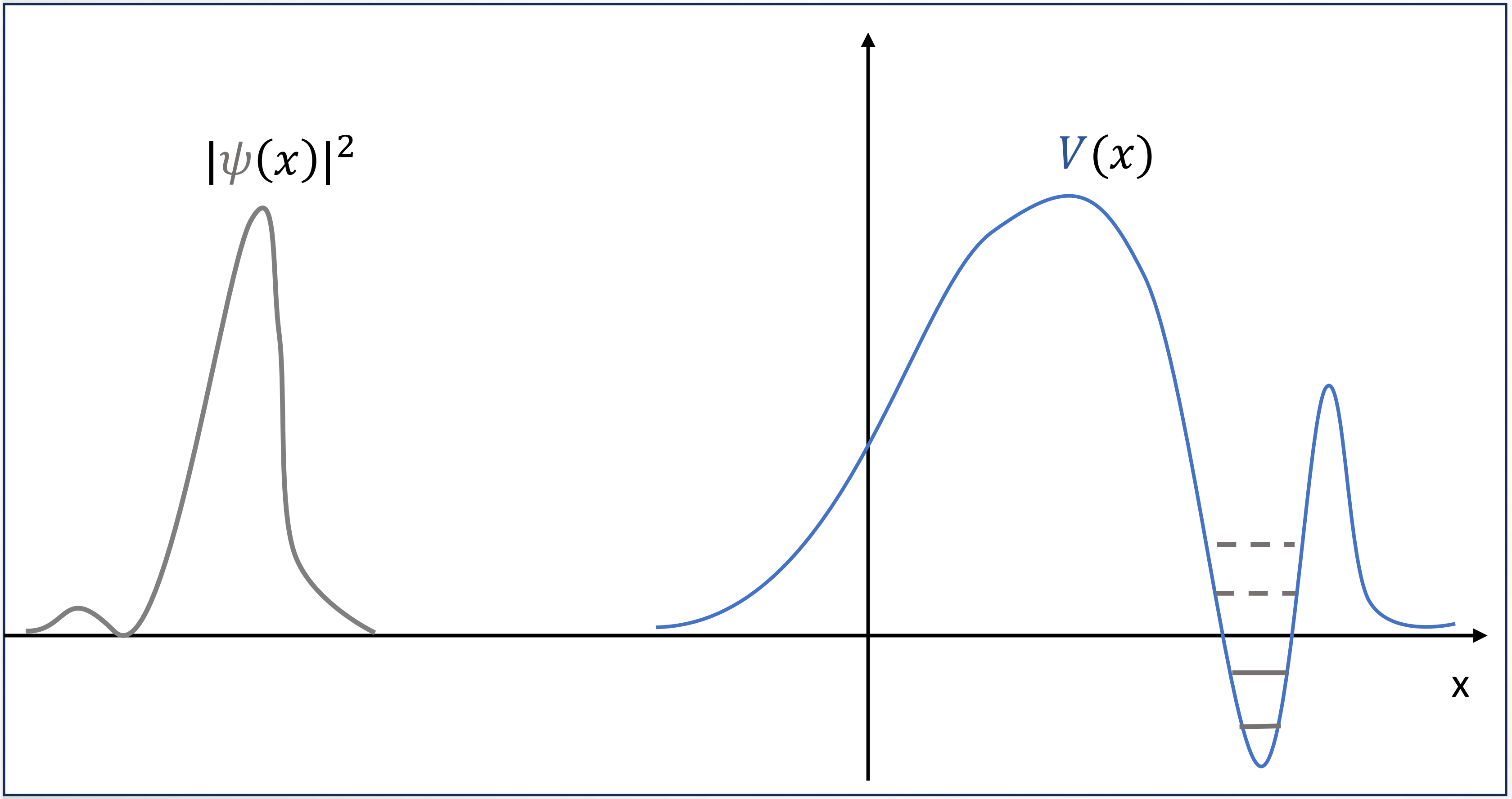
In other words, in the interaction picture

$$\langle\psi_{out}^{free}|S|\psi_{in}^{free}\rangle = \langle\psi_{out}|\psi_{in}\rangle = \lim_{\substack{t_-\to-\infty \\ t_+\to+\infty}} \langle\tilde{\psi}_{out}^{free}|U_I(t_+,t_-)|\tilde{\psi}_{in}^{free}\rangle$$

How the heck are relativistic scalar fields supposed not to see a static classical source? (non-relativistic particles see them!)

If formally corretct, it is complicated to work with wavefunctions, most people prefer the basis $\hat{P}_0^\mu |p\rangle = p^\mu|_{free} \, |p\rangle$

Staring from the S-matrix idea



$$\langle \psi_{out} | \psi_{in} \rangle = \langle \psi_{out}^{free} | S | \psi_{in}^{free} \rangle$$

In other words, in the interaction picture

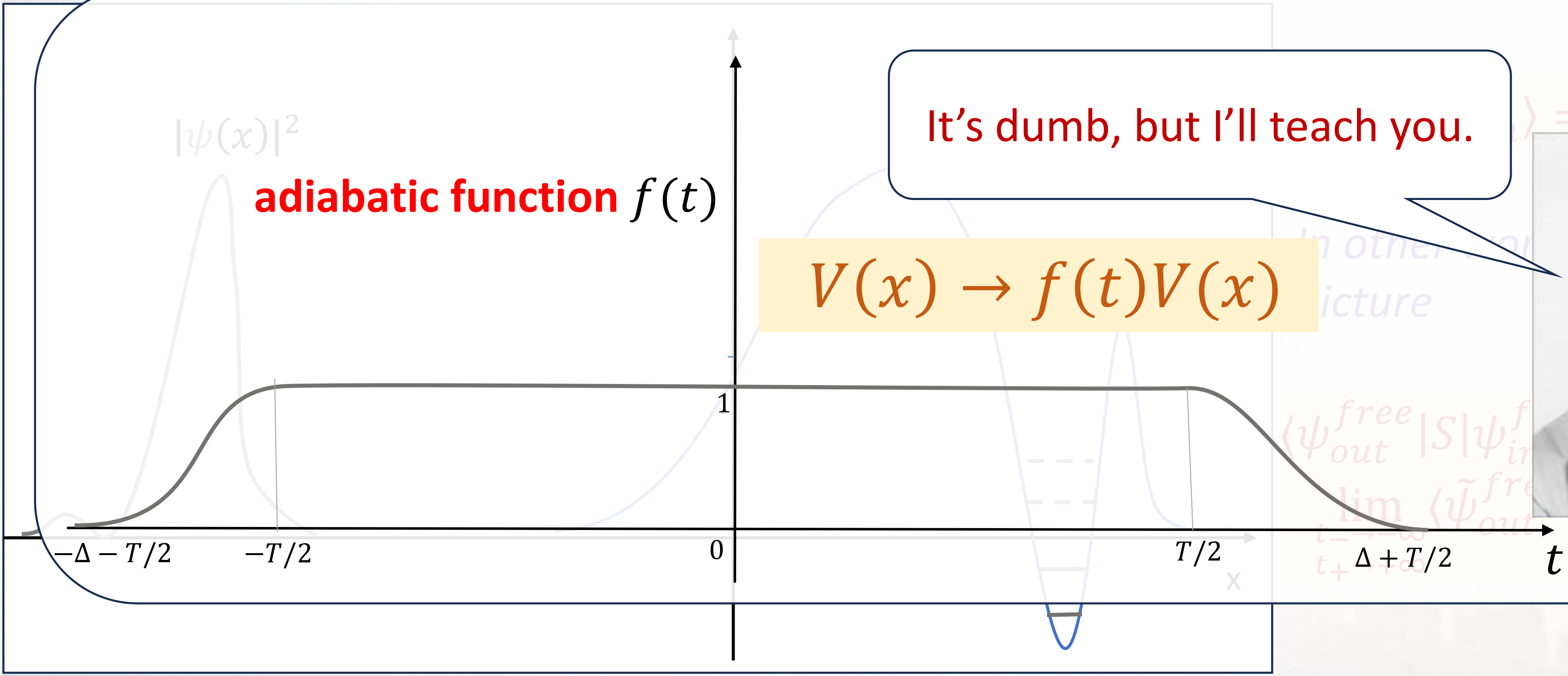
$$\langle \psi_{out}^{free} | S | \psi_{in}^{free} \rangle = \langle \psi_{out} | \psi_{in} \rangle = \lim_{\substack{t_- \to -\infty \\ t_+ \to +\infty}} \langle \tilde{\psi}_{out}^{free} | U_I(t_+, t_-) | \tilde{\psi}_{in}^{free} \rangle$$

How the heck are relativistic scalar fields supposed not to see a static classical source? (non-relativistic particles see them!)

How to “force” some free states in among the physical ones?

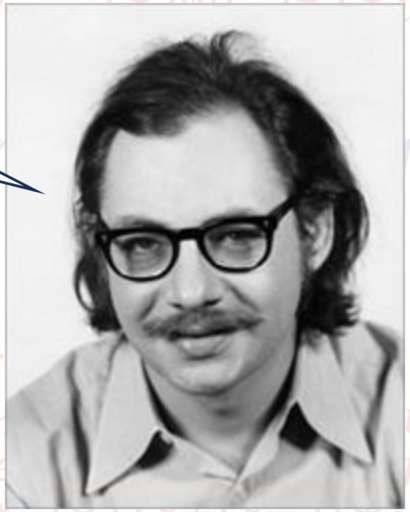
To make a good use of $U_I(t, t_0)$

Staring from the S-matrix idea



$$V(x) \rightarrow f(t)V(x)$$

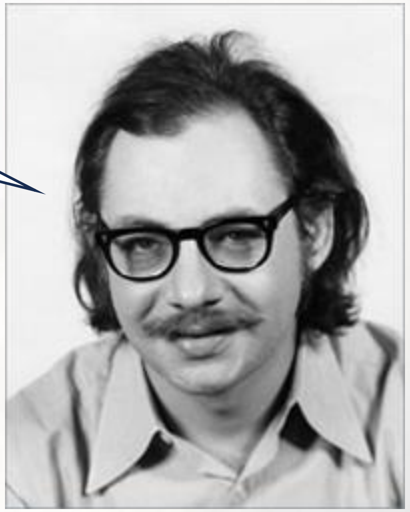
It’s dumb, but I’ll teach you.



How the heck are relativistic scalar fields supposed not to see a static classical source?(non-relativistic particles see them!)

Rather than from the Schrödinger equation and the action of the Hamiltonian on a generic state, one starts from the Klein-Gordon equation with a source

It's dumb, but I'll teach you.



$$[\Box + m^2]\,\hat{\phi}(t,\boldsymbol{x}) = g\,\rho(\boldsymbol{x})$$

As long as the state of the system is localized far away from the source, the only part of the field that can act upon it evolves as a free one $[\Box + m^2]\,\hat{\phi}_0(t,\boldsymbol{x}) = 0$

Since the quantum observable are built from the field $\hat{\phi}(x)$ (like the energy density), the situation is analogous to the single particle case (one expects asymptotically free states, as long as they are far enough).

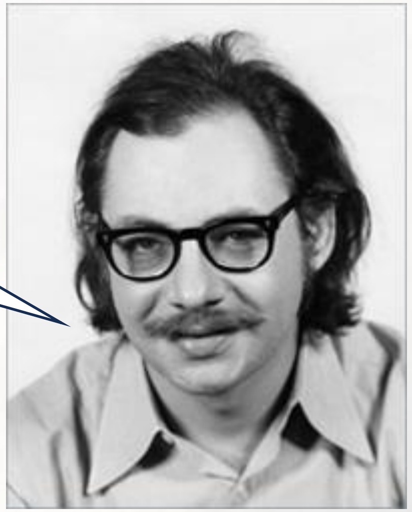
What could possibly go wrong?

How the heck are relativistic scalar fields supposed not to see a static classical source? (non-relativistic particles see them!)

Making use of the adiabatic switch in the (interaction picture) Hamiltonian, solving the Dyson series, using the Wick's theorem, factorizing and exponentiating the connected Wick's diagrams, one finally has

$$S \simeq \lim_{T\to\infty} U_I \simeq e^{-iE_0T}\ \hat{1}$$

I teach it at the beginning of the QFT course because it's simple, but a real physicist should do better!



This is consistent with the hypothesis that the interaction has the sole effect to add a time dependent phase $\psi \to e^{-i\,E_0 t}\psi$ to any state.

Such a minimal change would leave physics unaffected. It would change only the energy measurements (and related ones), but only as an off-set. No changes in relative energies measurements

In fact, it could be removed altogether by adding a counterterm to the Lagrangian, as usual.

Exact solution (with a generic source)

$$[\Box + m^2]\,\hat{\phi}_0(t, \boldsymbol{x}) = 0$$

The field equation can be solved with the Green functions, just not in their more familiar incarnation

$$[\Box + m^2]\,\hat{\phi}(x) = g\,\rho(x) \Rightarrow \hat{\phi}\,(x) = \hat{\phi}_0(x) + g\int d^4y\, G(x-y)\rho(y)$$

It is not uncommon to see the use of the retarded Green function (having a $\theta(t - y^0)$), but this ix incompatible with a time-independent source. The general solution reads

$$\hat{\phi}(x) = \hat{\phi}_0(x) + ig\int_{t_0}^{t} dy^0 \int d^3y\,\rho(y)\int \frac{d^3p}{(2\pi)^3 2E_{\mathbf{p}}}\left(e^{-ip\cdot(x-y)} - e^{ip\cdot(x-y)}\right)$$

It can be obtained splitting the time axis $(-\infty,\ \infty) = (-\infty, t_0] \cup [t_0, \infty)$, and using the advanced function in the $t \leq t_0$ region, and retarded function in the $t \geq t_0$ one

Exact solution (with a generic source)

Using the normalization

$$\hat{\phi}_0(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\boldsymbol{p}}}}\left[a_{\boldsymbol{p}}\, e^{-i\,p\cdot x} + h.c.\,\right]$$

Using one has

$$\hat{\phi}(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\boldsymbol{p}}}}\left[\left(a_{\boldsymbol{p}} + f(t,\boldsymbol{p})\right)\; e^{-i\,p\cdot x} + h.c.\,\right]$$

$$a_{\boldsymbol{p}}(t)$$

The additional term to the ladder operator is the regular function

$$f(t,\boldsymbol{p}) = i\,g\int_{t_0}^{t} dy^0 \int \frac{d^3y}{\sqrt{2E_{\boldsymbol{p}}}}\,\rho(y)\, e^{ip\cdot y}$$

Exact solution (with a generic source)

It is immediate to recognize the number operator

$$\hat{N}(t) = \int \frac{d^3p}{(2\pi)^3} \left[a_{\boldsymbol{p}}^{\dagger}(t) a_{\boldsymbol{p}}(t) \right]$$

Using one has

$$N_0(\mathrm{t}) = \langle 0|\hat{N}(t)|0\rangle = \int \frac{d^3p}{(2\pi)^3} \, |f^*(t,\boldsymbol{p}) f(t,\boldsymbol{p})|^2$$

In stark contrast with the expectations, the particle vacuum is not stable for a time-independent source

$$f(t,\boldsymbol{p}) = i\, g \int_{t_0}^{t} dy^0 \int \frac{d^3y}{\sqrt{2E_{\boldsymbol{p}}}} \, \rho(\boldsymbol{y}) \, e^{ip\cdot y} = g \, \frac{e^{iE_{\boldsymbol{p}}t} - e^{iE_{\boldsymbol{p}}t_0}}{\left(2E_{\boldsymbol{p}}^3\right)^{1/2}} \int d^3y \; \rho(\boldsymbol{y}) \, e^{-i\boldsymbol{p}\cdot y}$$

Exact solution (time-independent source)

$$f(t,\boldsymbol{p}) = i\,g\int_{t_0}^{t} dy^0 \int \frac{d^3y}{\sqrt{2E_{\boldsymbol{p}}}}\,\rho(\boldsymbol{y})\,e^{ip\cdot y} = g\,\frac{e^{iE_{\boldsymbol{p}}t}-e^{iE_{\boldsymbol{p}}t_0}}{(2E_{\boldsymbol{p}}^3)^{1/2}}\int d^3y\;\rho(\boldsymbol{y})\,e^{-i\boldsymbol{p}\cdot\boldsymbol{y}}$$

In terms of the source, the expected number of particles read

$$\tilde{\rho}(\boldsymbol{p}) = \int d^3y\;\rho(\boldsymbol{y})\,e^{-i\boldsymbol{p}\cdot\boldsymbol{y}}$$

$$N_0(\mathrm{t}) = g^2\int \frac{d^3p}{(2\pi)^3}\,\frac{4\,\sin^2(E_{\boldsymbol{p}}\Delta t/2)}{2\,E_{\boldsymbol{p}}^3}\,|\tilde{\rho}(\boldsymbol{p})|^2 = g^2\int \frac{d^3p}{(2\pi\,E_{\boldsymbol{p}})^3}\,|\tilde{\rho}(\boldsymbol{p})|^2\left[1-\ \cos(E_{\boldsymbol{p}}\Delta t)\ \right]$$

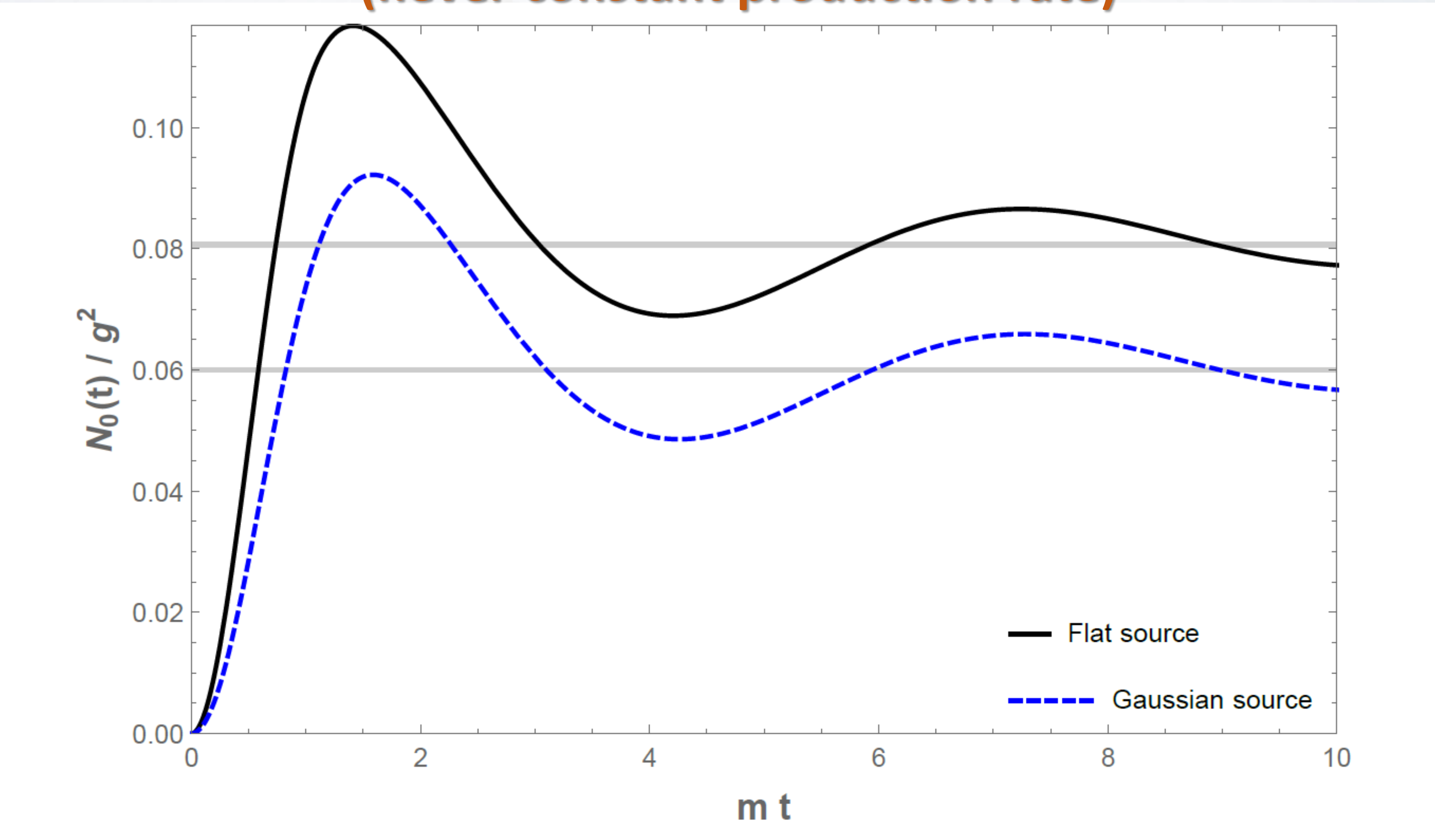
It never disappears, and the large volume limit of the asymptotic density depends on the shape

$$\lim_{L\to\infty}\lim_{t\to\infty}\frac{1}{L^3}N_0(t) = 2g^2m^3 \qquad \text{(flat source)},$$

$$\lim_{L\to\infty}\lim_{t\to\infty}\frac{1}{L^3}N_0(t) = \frac{g^2}{\sqrt{2}}m^3 \qquad \text{(Gaussian source)}.$$

Psrticles production

(never constant production rate)



Exact solution (with a generic source)

Thanks to the full exact expression $\hat{\phi}(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\boldsymbol{p}}}} \left[\left(a_{\boldsymbol{p}} + f(t, \boldsymbol{p}) \right) \; e^{-i\,p\cdot x} + h.c. \right]$

$$f(t, \boldsymbol{p}) = i\,g \int_{t_0}^{t} dy^0 \int \frac{d^3y}{\sqrt{2E_{\boldsymbol{p}}}}\, \rho(y)\, e^{ip\cdot y}$$

$$a_{\boldsymbol{p}}(t)$$

and the properties of the time-dependent ladder operators, one can fully solve for the partial wave amplitudes

$$\psi(t; \boldsymbol{p}_1, \cdots, \boldsymbol{p}_n) = \langle \boldsymbol{p}_1, \cdots, \boldsymbol{p}_n | U(t, t_0) | \psi \rangle$$

for a generic initial state $|\psi\rangle$

Exact solution (with a generic source)

$$\hat{\phi}(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\left[\left(a_p + f(t,\mathbf{p})\right)\ e^{-i\,p\cdot x} + h.c.\ \right]$$

$$f(t,\mathbf{p}) = i\,g\int_0^t dy^0 \int \frac{d^3y}{\sqrt{2E_\mathbf{p}}}\,\rho(y)\,e^{ip\cdot y}$$

$a_\mathbf{p}(t)$

Indeed

$$a_\mathbf{p}(t) = \frac{i}{\sqrt{2E_\mathbf{p}}}\int d^3x\; e^{iE_\mathbf{p}t}\,e^{-i\mathbf{p}\cdot\mathbf{x}}\left[\hat{\Pi}(t,\mathbf{x}) - iE_\mathbf{p}\,\hat{\phi}(t,\mathbf{x})\right],$$

and therefore

$$\begin{aligned}
a_\mathbf{p}(t) &= \frac{i}{\sqrt{2E_\mathbf{p}}}\int d^3x\; e^{iE_\mathbf{p}t}\,e^{-i\mathbf{p}\cdot\mathbf{x}}\left[U^{-1}(t,0)\,\hat{\Pi}(0,\mathbf{x})\,U(t,0) - iE_\mathbf{p}\,U^{-1}(t,0)\,\hat{\phi}(0,\mathbf{x})\,U(t,0)\right] \\
&= U^{-1}(t,0)\left\{\frac{i}{\sqrt{2E_\mathbf{p}}}\int d^3x\; e^{iE_\mathbf{p}t}\,e^{-i\mathbf{p}\cdot\mathbf{x}}\left[\hat{\Pi}(0,\mathbf{x}) - iE_\mathbf{p}\,\hat{\phi}(0,\mathbf{x})\right]\right\}U(t,0) \\
&= U^{-1}(t,0)\left\{e^{iE_\mathbf{p}t}\,a_\mathbf{p}\right\}U(t,0)\ .
\end{aligned}$$

the extra phase can be written in terms of the free propagator $U_0(t,0)$, as usual for the ladder operators

Exact solution (with a generic source)

$$\hat{\phi}(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\boldsymbol{p}}}}\left[\left(a_{\boldsymbol{p}} + f(t,\boldsymbol{p})\right)\ e^{-i\,p\cdot x} + h.c.\ \right]$$

$a_{\boldsymbol{p}} \to a_{\boldsymbol{p}}(t)$

$$f(t,\boldsymbol{p}) = i\,g\int_0^t dy^0 \int \frac{d^3y}{\sqrt{2E_{\boldsymbol{p}}}}\,\rho(y)\,e^{ip\cdot y}$$

The time-dependent ladder operators evolve with the interaction picture propagator

$$a_{\boldsymbol{p}}(t) = U_I^{-1}(t,0)\,a_{\boldsymbol{p}}\,U_I(t,0)$$

time-dependent basis of particles eigenstates

$$|0\rangle_t = U_I^{-1}(t,0)|0\rangle \quad \Rightarrow \quad a_{\mathbf{p}}(t)|0\rangle_t = \left(U_I^{-1}(t,0)\,a_{\mathbf{p}}\,U_I(t,0)\right)U_I^{-1}(t,0)|0\rangle$$
$$= U_I^{-1}(t,0)\left(a_{\mathbf{p}}|0\rangle\right) = 0,$$

$$|\mathbf{p}_1,\cdots,\mathbf{p}_n\rangle_t = U_I^{-1}(t,0)|\mathbf{p}_1,\cdots,\mathbf{p}_n\rangle \quad \Rightarrow$$
$$a^\dagger_{\mathbf{p}}(t)\,a_{\mathbf{p}}(t)\,|\mathbf{p}_1,\cdots,\mathbf{p}_n\rangle_t = U_I^{-1}(t,0)\left(a^\dagger_{\mathbf{p}}\,a_{\mathbf{p}}\,|\mathbf{p}_1,\cdots,\mathbf{p}_n\rangle\right).$$

Exact solution (with a generic source)

General decomposition of a state

$$\psi(t;\boldsymbol{p}_1,\cdots,\boldsymbol{p}_n) = \langle \boldsymbol{p}_1,\cdots,\boldsymbol{p}_n|U(t,0)|\psi\rangle = \langle \boldsymbol{p}_1,\cdots,\boldsymbol{p}_n|U_0(t,0)\, U_I(t,0)|\psi\rangle = \prod_{j=1}^{n} e^{-i\,E_{\boldsymbol{p}_j}t}\,\langle \boldsymbol{p}_1,\cdots,\boldsymbol{p}_n|U_I(t,0)|\psi\rangle$$

$$= e^{-i\left(\sum_j E_{\boldsymbol{p}_j}\right)t}\;{}_t\langle \boldsymbol{p}_1,\cdots,\boldsymbol{p}_n|U(t,0)|\psi\rangle$$

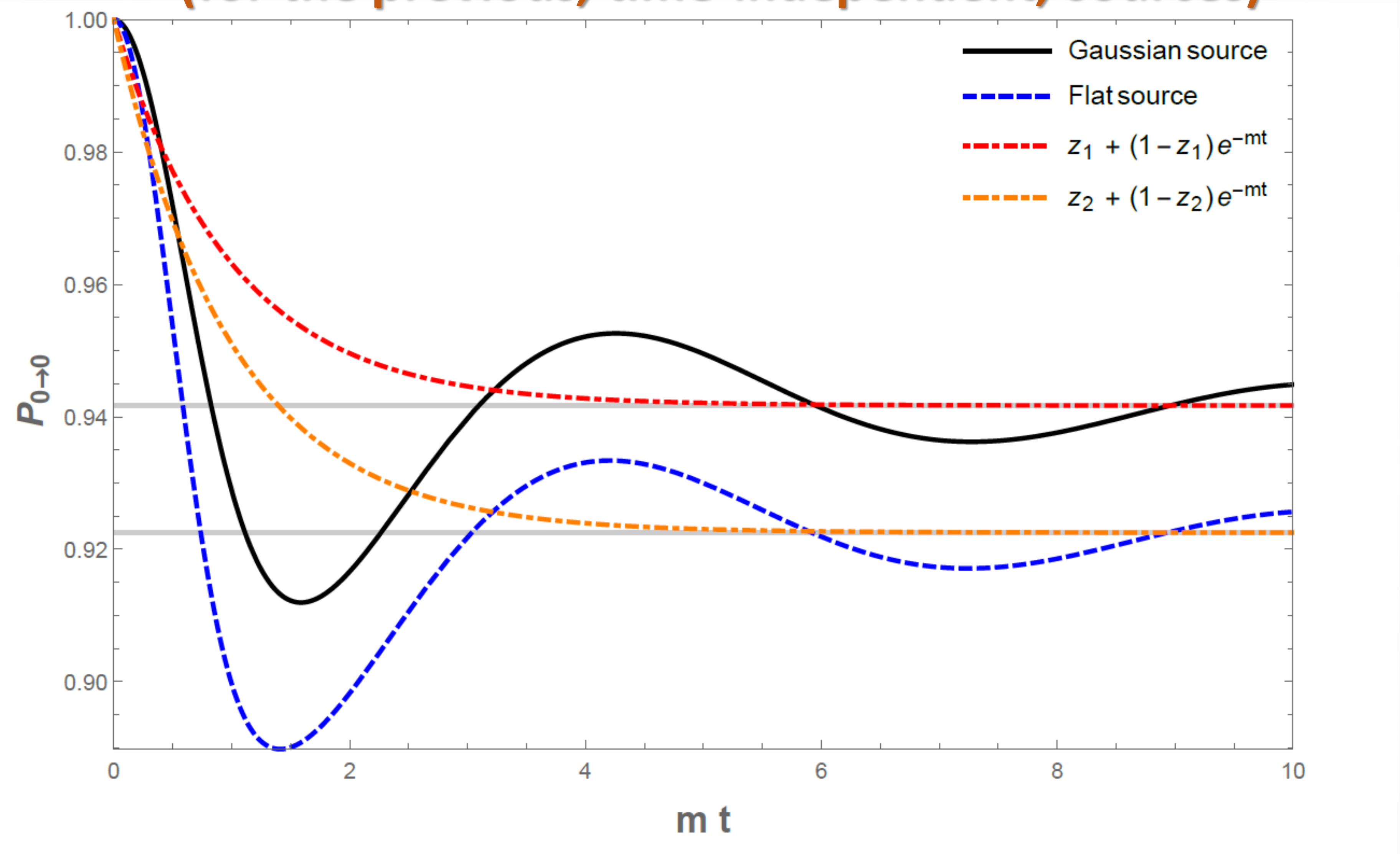
the shift in the time dependent ladder operator does not change their commutation relations $\Rightarrow\ {}_t\langle p_1,\cdots p_n|k_1,\cdots,k_n\rangle$, *in particular*

$$\psi_0(t;\boldsymbol{p}_1,\cdots,\boldsymbol{p}_n) = \langle \boldsymbol{p}_1,\cdots,\boldsymbol{p}_n|U(t,0)|0\rangle = \frac{\mathrm{e}^{-\frac{1}{2}N_0(t)+i\phi(t)}}{\sqrt{n!}}\prod_j\left[e^{-iE_{\boldsymbol{p}_j}t}\, f\left(t,\boldsymbol{p}_j\right)\sqrt{2E_{\boldsymbol{p}_j}}\right]$$

The probability of the vacuum to remain is e^{-N_0}, not an exponential decay law.

Particular case: vacuum decay

(for the previous, time-independent, sources)



What went wrong in the past?

Besides the field equations the Lagrangian density can gives the four-momentum operators

$$\frac{1}{2}\partial_\mu \hat{\phi}\, \partial^\mu \hat{\phi} - \frac{1}{2} m^2\, \hat{\phi}^2 + g\, \rho\, \hat{\phi}$$

$$\hat{P}^\mu = \int d^3x\ \hat{T}^{0\mu} = \begin{cases} \hat{H} = \int d^3x\ \hat{T}^{00} = \int \frac{d^3p}{(2\pi)^3}\, E_{\boldsymbol{p}} \left[a_{\boldsymbol{p}}^\dagger + f^*(t,\boldsymbol{p}) - \frac{i}{E_{\boldsymbol{p}}}\partial_t f^*(t,\boldsymbol{p})\right]\left[a_{\boldsymbol{p}} + f\,(t,\boldsymbol{p}) + \frac{i}{E_{\boldsymbol{p}}}\partial_t f\,(t,\boldsymbol{p})\right] \\ \hat{P}^i = \int d^3x\ \hat{T}^{0i} = \int \frac{d^3p}{(2\pi)^3}\, p^i \left[a_{\boldsymbol{p}}^\dagger + f^*(t,\boldsymbol{p})\ \right]\left[a_{\boldsymbol{p}} + f\,(t,\boldsymbol{p})\ \right] = \int \frac{d^3p}{(2\pi)^3}\, p^i a_{\boldsymbol{p}}^\dagger(t) a_{\boldsymbol{p}}(t) \end{cases}$$

The energy eigenstates are coherent states of the particle and momentum eigenstates

No asymptotically free states! In fact, no regular vacuum

$$\hat{P}^\mu\, |\Omega\rangle = 0\ \forall\ \mu$$

But the arguments for the free states seemed ok

Do we have something similar?

$$|\psi_1\rangle = \int \frac{d^3k}{(2\pi)^3 2E_{\boldsymbol{k}}} \psi(\boldsymbol{k})|\boldsymbol{k}\rangle$$

The full evolution of states is an interference pattern between the decaying vacuum, absorption amplitudes, and the original wavefunction

$$\langle \mathbf{p}_1 \cdots \mathbf{p}_n | U(t,0) |\psi_1\rangle = -\psi_0(t;\mathbf{p}_1 \cdots \mathbf{p}_n) \int \frac{d^3k}{(2\pi)^3\sqrt{2E_{\mathbf{k}}}} f^*(t,\mathbf{k})\,\psi_1(\mathbf{k}) + \frac{1}{\sqrt{n}} \sum_{j=1}^{n} \left[\psi_1(\mathbf{p}_j)\, e^{-iE_{\mathbf{p}_j}t}\ \psi_0(t;\mathbf{p}_1,\cdots \mathbf{p}_{j-1},\mathbf{p}_{j+1},\cdots,\mathbf{p}_n)\right].$$

The one particle partial wave reads

$$\left[\mathcal{A}(t)\,\psi_0(t;\mathbf{p}) + \psi_0(t)\,e^{-iE_{\mathbf{p}}t}\psi_i(\mathbf{p})\right] \frac{\exp\left[\frac{1}{2}N_0(t)\right]}{\sqrt{|\mathcal{A}(t)|^2\left[N_0(t)-2\right]+1}}$$

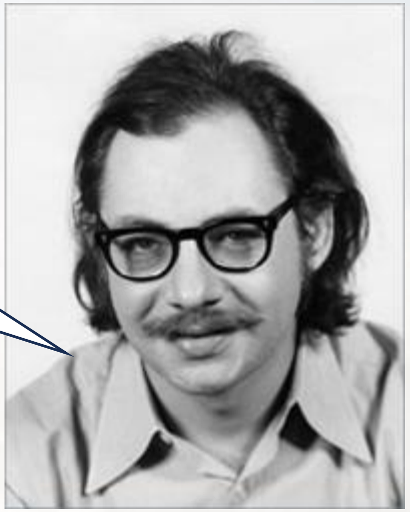
with

$$\mathcal{A}(t) = -\int \frac{d^3k}{(2\pi)^3\sqrt{2E_{\mathbf{k}}}} f^*(t,\mathbf{k})\,\psi_1(\mathbf{k}) = -i\,g\int_0^t dy^0 \int d^3y \int \frac{d^3k}{(2\pi)^3 2E_{\mathbf{k}}}\,\rho(\mathbf{y})\,e^{-ip\cdot y}\psi_1(\mathbf{k})$$
$$= -i\int_0^t dy^0 \int d^3y\; g\,\rho(\mathbf{y})\,\psi_1(y).$$

But the arguments for the free states seemed ok

Do we have something similar?

Can we see if an alien civilization in the Andromeda galaxy turned on a classical source from this interference pattern?



The full evolution of states is an interference pattern between the decaying vacuum, absorption amplitudes, and the original wavefunction

$$\psi_0(t;\boldsymbol{p}_1,\cdots,\boldsymbol{p}_n)=\langle\boldsymbol{p}_1,\cdots,\boldsymbol{p}_n|U(t,0)|0\rangle=\frac{\mathrm{e}^{-\frac{1}{2}N_0(t)+i\phi(t)}}{\sqrt{n!}}\prod_j\left[e^{-iE_{\boldsymbol{p}_j}t}f(t,\boldsymbol{p}_j)\sqrt{2E_{\boldsymbol{p}_j}}\right]$$

$$\langle\mathbf{p}_1\cdots\mathbf{p}_n|\,U(t,0)\,|\psi_1\rangle=-\psi_0(t;\mathbf{p}_1\cdots\mathbf{p}_n)\int\frac{d^3k}{(2\pi)^3\sqrt{2E_{\mathbf{k}}}}\,f^*(t,\mathbf{k})\,\psi_1(\mathbf{k})$$
$$+\frac{1}{\sqrt{n}}\sum_{j=1}^{n}\left[\psi_1(\mathbf{p}_j)\,e^{-iE_{\mathbf{p}_j}t}\,\psi_0(t;\mathbf{p}_1,\cdots\mathbf{p}_{j-1},\mathbf{p}_{j+1},\cdots,\mathbf{p}_n)\right].$$

Conclusions and outlook

- Despite the highly held expectations, the interaction is not trivial for time-independent classical sources.
- Exact solutions of the Schwinger like vacuum decay, no constant particle production rate.
- Non perturbative source of particles from a medium, especially important for suppressed perturbative processes.

Thank you for your attention!