Correlations and local production of open charm

Outline

- Introduction and motivations: limits of the standard picture of heavy-ion collisions
- Unclear prediction over correlations: conflicting point of views
- Open charm correlations as a way to settle the more appropriate phenomenological extensions

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M Gazdzicki, D Kikoła, I Pidhurskyi, Leonardo Tinti

Uniwersytet Jana Kochanowskiego w Kielcach

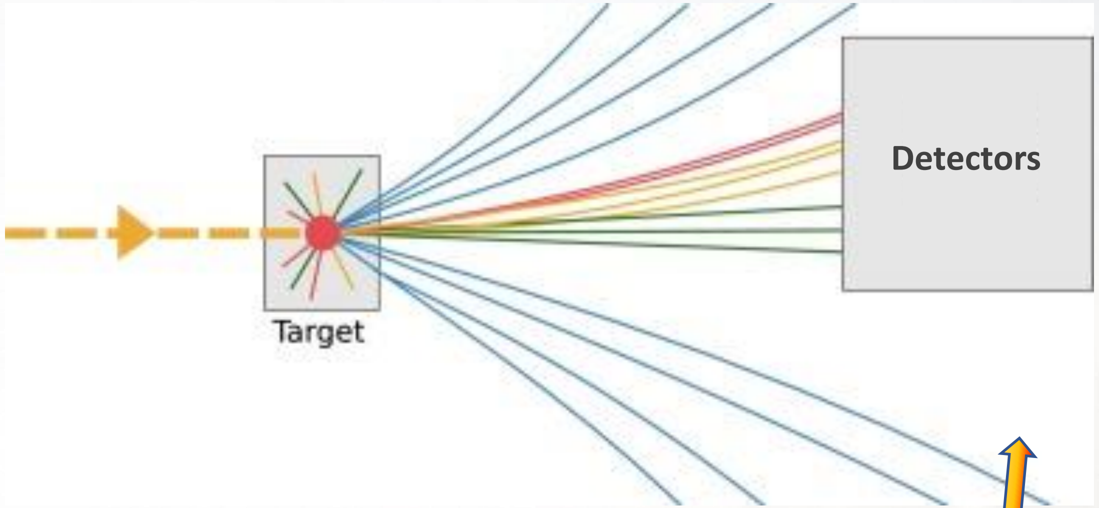
Heavy-ion collisions

What we should do (If we could)

- Initial states (sharply peaked gaussians in momentum space)

$\left(\propto e^{-\frac{(p-p_0)^2}{2\sigma^2}-ip\cdot(x-x_0)}\ldots\right)$

$| \Psi_{in} \rangle$



- Evolution operator (from the initial time t_i to the final one t_f)

$$\hat{U}(t_f, t_i) = e^{-i\hat{H}(t_f - t_i)}$$

$\propto S$ matrix

- Projection with some final states (momentum states of the final particles)

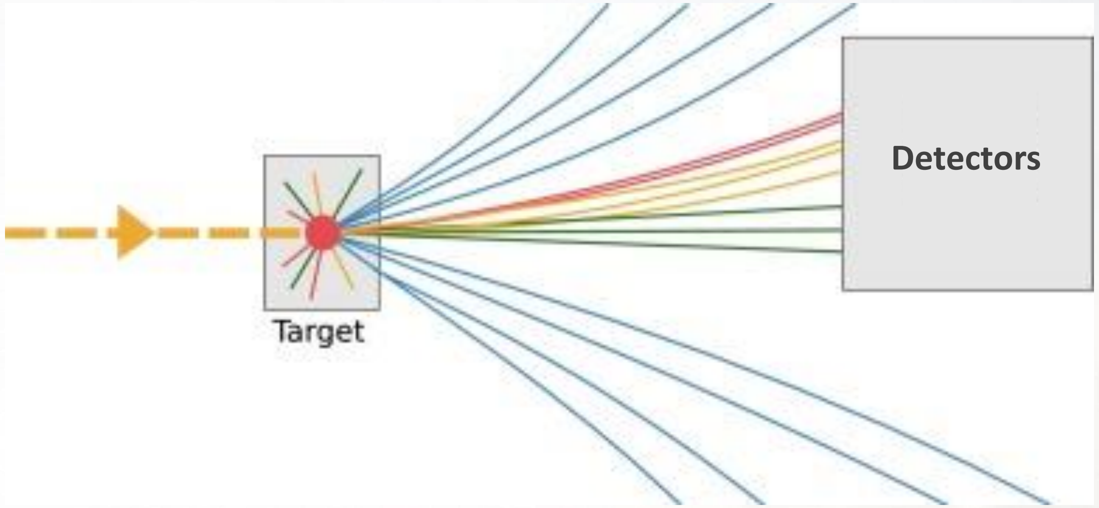
$$P(out|in) = \left|\langle \Psi_{out}|\hat{U}(t_f, t_i)|\Psi_{in}\rangle\right|^2$$

Heavy-ion collisions

What we should do (If we could)

- Schrodinger picture(interference, fluctuations all taken into account)

$$|\Psi_S(t)\rangle = \sum_{\boldsymbol{n}} e^{-i(t-t_0)E_n} \alpha_{\boldsymbol{n}} |\boldsymbol{n}\rangle$$



- Expectations values (for any observable)

$$O = tr\left(\rho \hat{O}\right) = tr\left(|\Psi_S\rangle\langle\Psi_S|\,\hat{O}\right)$$

- With $\rho = |\Psi_S(t)\rangle\langle\Psi_S(t)| = \sum_{\boldsymbol{n},\boldsymbol{m}} e^{-i(t-t_0)(E_n-E_m)} \alpha^*_{\boldsymbol{n}} \alpha_{\boldsymbol{m}} |\boldsymbol{n}\rangle\langle\boldsymbol{m}|$ Too complicated!

Heavy-ion collisions

What we did (thermal model)

- Substitute the (inconveniently complicated) exact state

$$\rho = |\Psi_S(t)\rangle\langle\Psi_S(t)| = \sum_{\boldsymbol{n},\boldsymbol{m}} e^{-i(t-t_0)(E_n-E_m)}\alpha_{\boldsymbol{n}}^*\alpha_{\boldsymbol{m}}|\boldsymbol{n}\rangle\langle\boldsymbol{m}|$$

- with the simpler, diagonal, mixed state (RPA can partially account for that)

$$\rho \simeq \sum_{\boldsymbol{n}} P_{\boldsymbol{n}}|\boldsymbol{n}\rangle\langle\boldsymbol{n}|$$

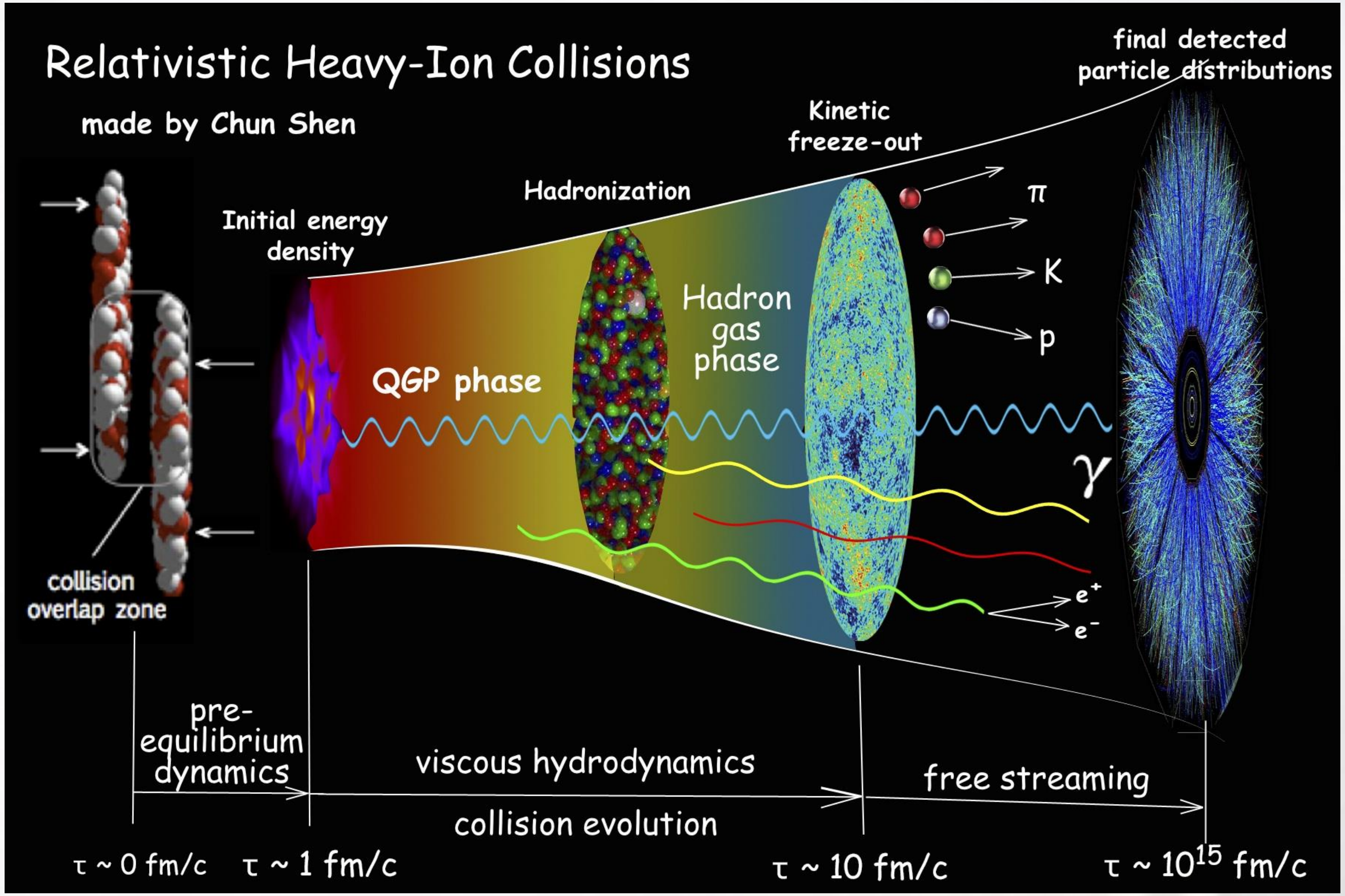
- Final states approximately free, we know the equilibrium state (microcanonical, canonical , etc.) and we can compute expectation values

$$O \simeq tr\left(\rho_{eq.}\hat{O}\right)\Big|_{\text{free}}$$

Heavy-ion collisions

What we do (now, mostly)

- Initial conditions (Monte Carlo Glauber, color glass condensate, etc…)
- Pre-hydro smoothening (gaussians, parton free-streaming, etc…)
- Hydrodynamics (ideal, second-order, aHydro, etc…)
- Hadronization (direct freeze-out or rescattering)



Comparisons between theory and experiments

What we compute (expectation values)

$$T^{\mu\nu}(x) = tr\left(\rho\, \hat{T}^{\mu\nu}(x)\right)$$

$$J_B^{\mu}(x) = tr\left(\rho\, \hat{J}_B^{\mu}(x)\right)$$

The (approximate) evolution is a closed set of equations, for each subset

What we measure

$$\frac{dN}{d^3p} \qquad \frac{d\bar{N}}{d^3p}$$

Spectra (momentum space), this is ok… but also other things

It is important to translate from one picture to the other in the appropriate way

A brief look at relativistic kinetic theory

The relativistic Boltzmann equation

$$\int d^4p\, 2\theta(p_0)\delta(p^2 - m^2) = \int \frac{d^3p}{E_{\boldsymbol{p}}}$$

$$p \cdot \partial f(x, \boldsymbol{p}) = C[f, \bar{f}]$$
$$p \cdot \partial \bar{f}(x, \boldsymbol{p}) = \bar{C}[f, \bar{f}]$$

Well defined stress-energy tensor and baryon current

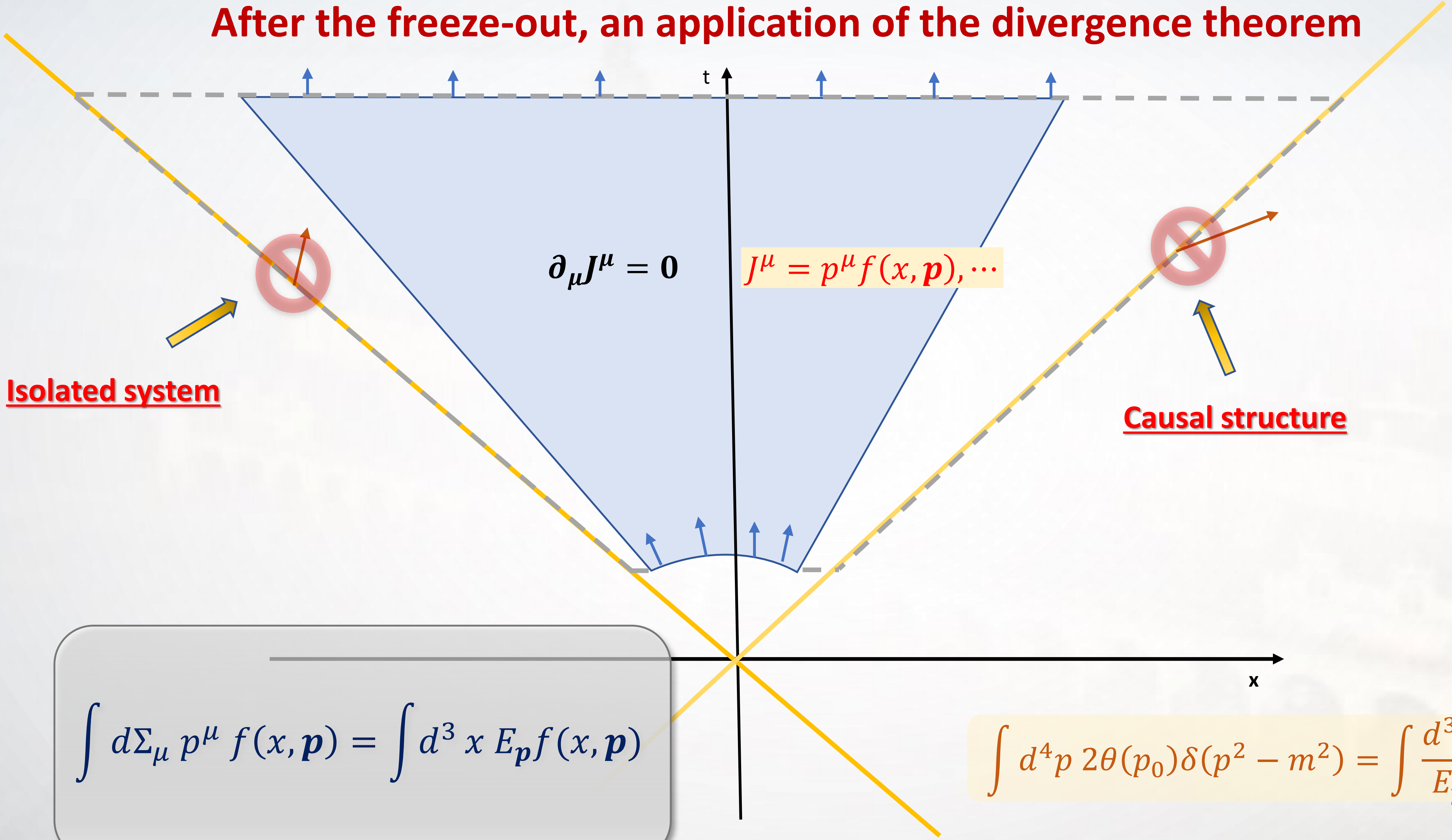
$$T^{\mu\nu}(x) = \frac{g_S}{(2\pi)^3} \int \frac{d^3p}{E_{\boldsymbol{p}}}\, p^\mu p^\nu \left(f(x, \boldsymbol{p}) + \bar{f}(x, \boldsymbol{p})\right)$$

$$J_B^\mu(x) = \frac{g_S}{(2\pi)^3} \int \frac{d^3p}{E_{\boldsymbol{p}}}\, p^\mu \left(f(x, \boldsymbol{p}) - \bar{f}(x, \boldsymbol{p})\right)$$

A bridge between hydro and spectra (but not fluctuations and correlations) in momentum space

Connection with the spectra

After the freeze-out, an application of the divergence theorem



$$\int d\Sigma_\mu \, p^\mu \, f(x, \boldsymbol{p}) = \int d^3x \, E_{\boldsymbol{p}} f(x, \boldsymbol{p})$$

$$\int d^4p \, 2\theta(p_0)\delta(p^2 - m^2) = \int \frac{d^3p}{E_{\boldsymbol{p}}}$$

Connection with the spectra

One-particle observable, can we use it for more?

$$\int d^4p\, 2\theta(p_0)\delta(p^2 - m^2) = \int \frac{d^3p}{E_{\boldsymbol{p}}}$$

$$\int \frac{d^3p}{(2\pi)^3}\left(\int d^3x\ f(x,\boldsymbol{p})\right) = N = \int \frac{d^3p}{(2\pi)^3}\langle a^\dagger(\boldsymbol{p})a(\boldsymbol{p})\rangle$$

$$\int \frac{d^3p}{(2\pi)^3}\left(\int d^3x\ \bar{f}(x,\boldsymbol{p})\right) = \bar{N} = \int \frac{d^3p}{(2\pi)^3}\langle b^\dagger(\boldsymbol{p})b(\boldsymbol{p})\rangle$$

$$\begin{cases} E_{\boldsymbol{p}} \int \frac{d^3x}{(2\pi)^3} f(x,\boldsymbol{p}) = E_{\boldsymbol{p}} \frac{dN}{d^3p} = \frac{1}{(2\pi)^3}\langle a^\dagger(\boldsymbol{p})a(\boldsymbol{p})\rangle \\ E_{\boldsymbol{p}} \int \frac{d^3x}{(2\pi)^3} \bar{f}(x,\boldsymbol{p}) = E_{\boldsymbol{p}} \frac{d\bar{N}}{d^3p} = \frac{1}{(2\pi)^3}\langle b^\dagger(\boldsymbol{p})b(\boldsymbol{p})\rangle \end{cases}$$

After all, both at the classical and quantum level, two physical interpretations:

- *probability density of the momentum of a single particle*
- *average number of particles in a single momentum cell*

Connection with the spectra

The distribution function as both a one-particle and a multi-particle information carier

From the 6N+1 dimensional phase space $\rho(\boldsymbol{x}_1 \cdots \boldsymbol{x}_N, \boldsymbol{p}_1 \cdots \boldsymbol{p}_N, t)$

the Liouville prescription

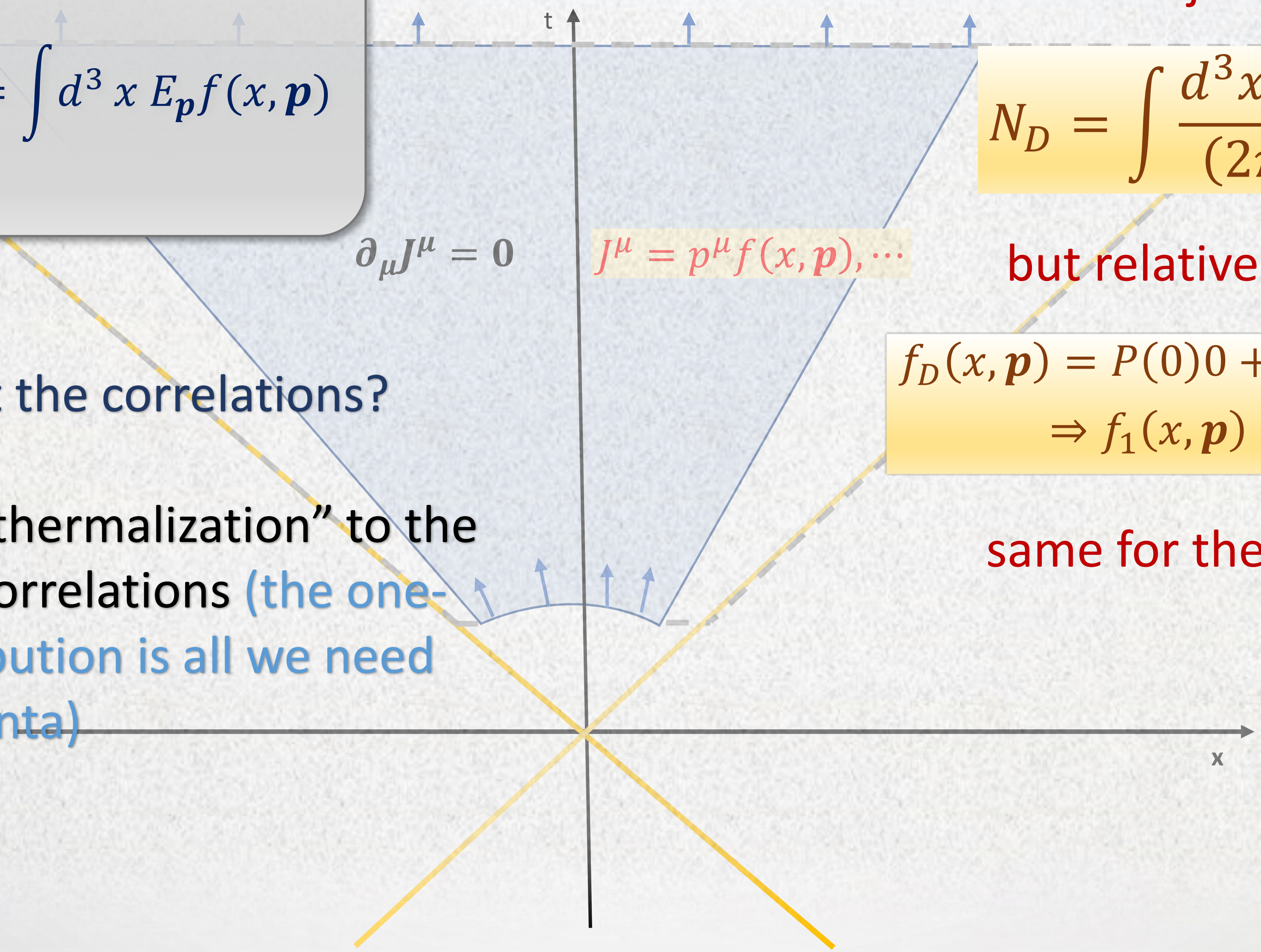
$$f(x, \boldsymbol{p}) = \sum_i \int \{[dp]^N - \boldsymbol{p}_i\} \rho(\boldsymbol{x}_1 \cdots \boldsymbol{x}_N, \boldsymbol{p}_1 \cdots \boldsymbol{p}_N, t)$$

similarly, in the quantum case, because of the commutation relations

$$\langle a^\dagger(\boldsymbol{p}) a(\boldsymbol{p}) \rangle \propto \sum_i \int \{[dp]^N - \boldsymbol{p}_i\} \, |\psi(\boldsymbol{p}_1 \cdots \boldsymbol{p}_n)|^2$$

Charm from the medium

$$\int d\Sigma_\mu \, p^\mu \, f(x,\boldsymbol{p}) = \int d^3 x \, E_{\boldsymbol{p}} f(x,\boldsymbol{p})$$



Some adjustments required

$$N_D = \int \frac{d^3x d^3p}{(2\pi)^3} f_D(x,\boldsymbol{p}) < \mathbf{1}$$

but relatively simple

$$f_D(x,\boldsymbol{p}) = P(0)0 + P(1) f_1(x,\boldsymbol{p}) + \cdots$$
$$\Rightarrow f_1(x,\boldsymbol{p}) \simeq \frac{f_D(x,\boldsymbol{p})}{P(1)}$$

same for the antiparticle

What about the correlations?

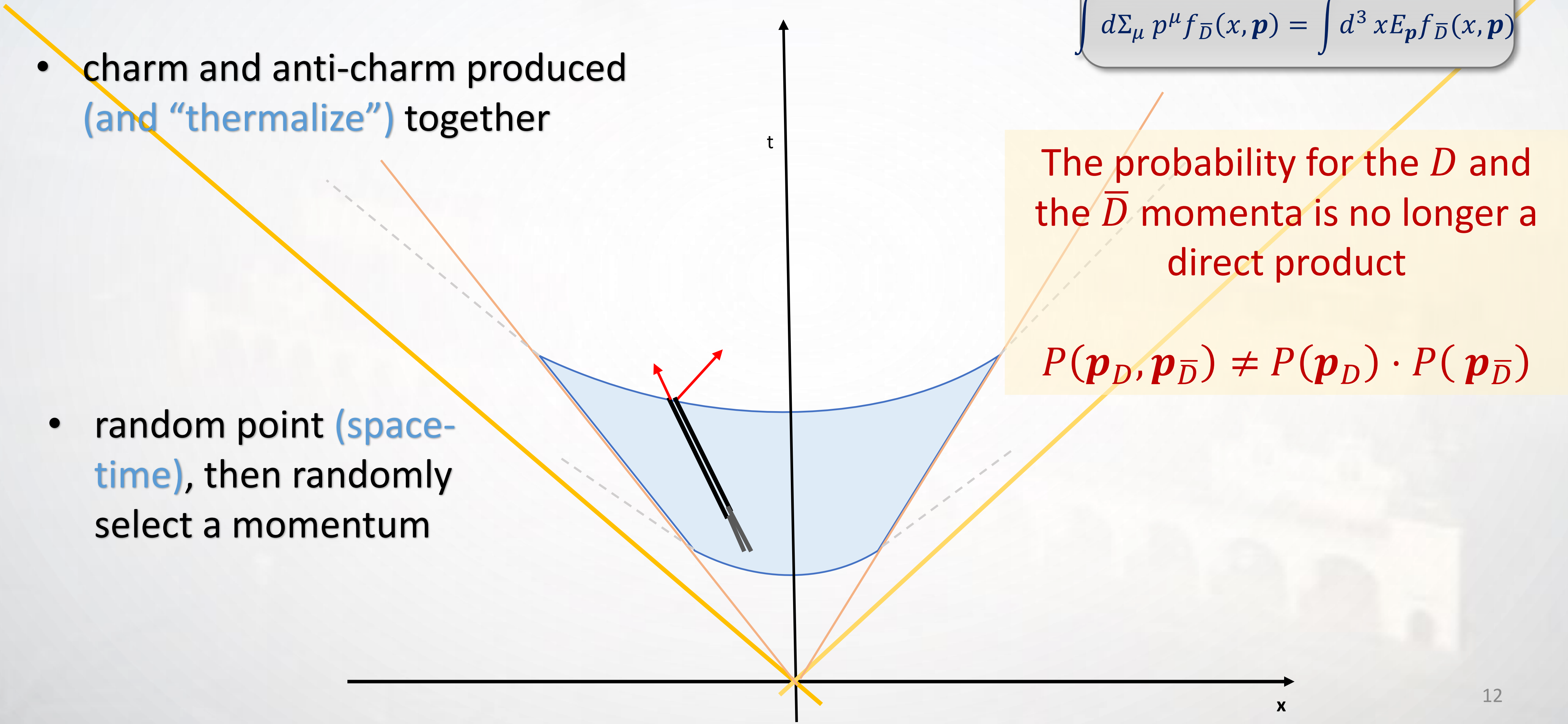
- First option: "thermalization" to the extreme, no correlations (the one-particle distribution is all we need for the momenta)

Charm from the medium

Similar idea, still consistent with the spectra prescription

$$\int d\Sigma_\mu \, p^\mu \, f_D(x,\mathbf{p}) = \int d^3x E_\mathbf{p} f_D(x,\mathbf{p})$$
$$\int d\Sigma_\mu \, p^\mu f_{\bar{D}}(x,\mathbf{p}) = \int d^3x E_\mathbf{p} f_{\bar{D}}(x,\mathbf{p})$$

- charm and anti-charm produced (and "thermalize") together
- random point (space-time), then randomly select a momentum



The probability for the D and the $\bar{D}$ momenta is no longer a direct product

$$P(\mathbf{p}_D,\mathbf{p}_{\bar{D}}) \neq P(\mathbf{p}_D)\cdot P(\mathbf{p}_{\bar{D}})$$

Charm from the medium

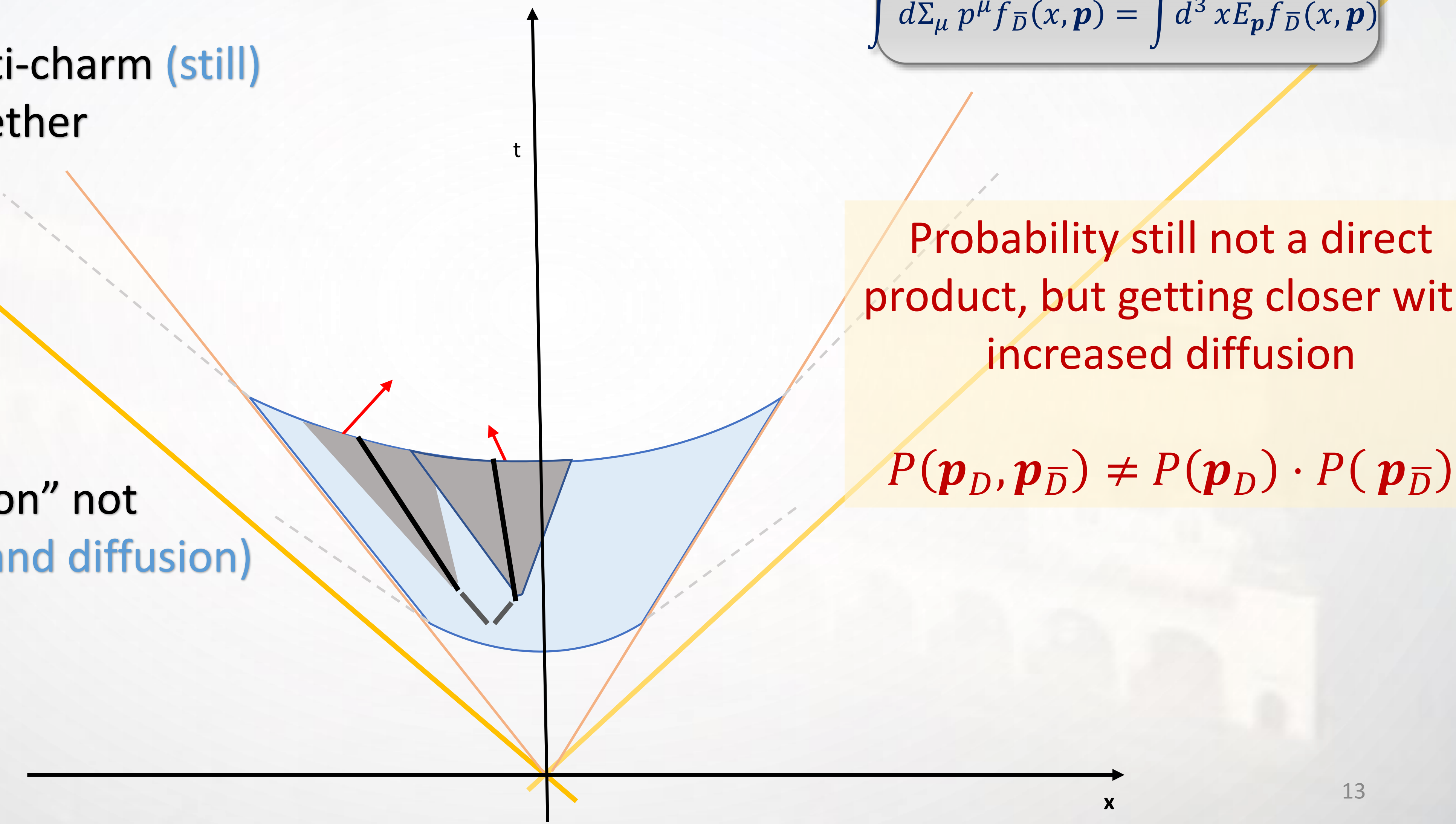
Intermediate (non-unique) picture

- charm and anti-charm (still) produced together

- "thermalization" not immediate (and diffusion)

$$\int d\Sigma_\mu \, p^\mu \, f_D(x,\mathbf{p}) = \int d^3x \, E_{\mathbf{p}} f_D(x,\mathbf{p})$$

$$\int d\Sigma_\mu \, p^\mu f_{\bar{D}}(x,\mathbf{p}) = \int d^3x E_{\mathbf{p}} f_{\bar{D}}(x,\mathbf{p})$$



Probability still not a direct product, but getting closer with increased diffusion

$$P(\mathbf{p}_D, \mathbf{p}_{\bar{D}}) \neq P(\mathbf{p}_D) \cdot P(\mathbf{p}_{\bar{D}})$$

Some estimates (from a rough model)

Hubble flow (spherically symmetric)

$$\frac{dx^\mu}{d\tau} = u^\mu = \frac{x^\mu}{\tau}$$

Maximum radius $\mathrm{R} = 6$ fm

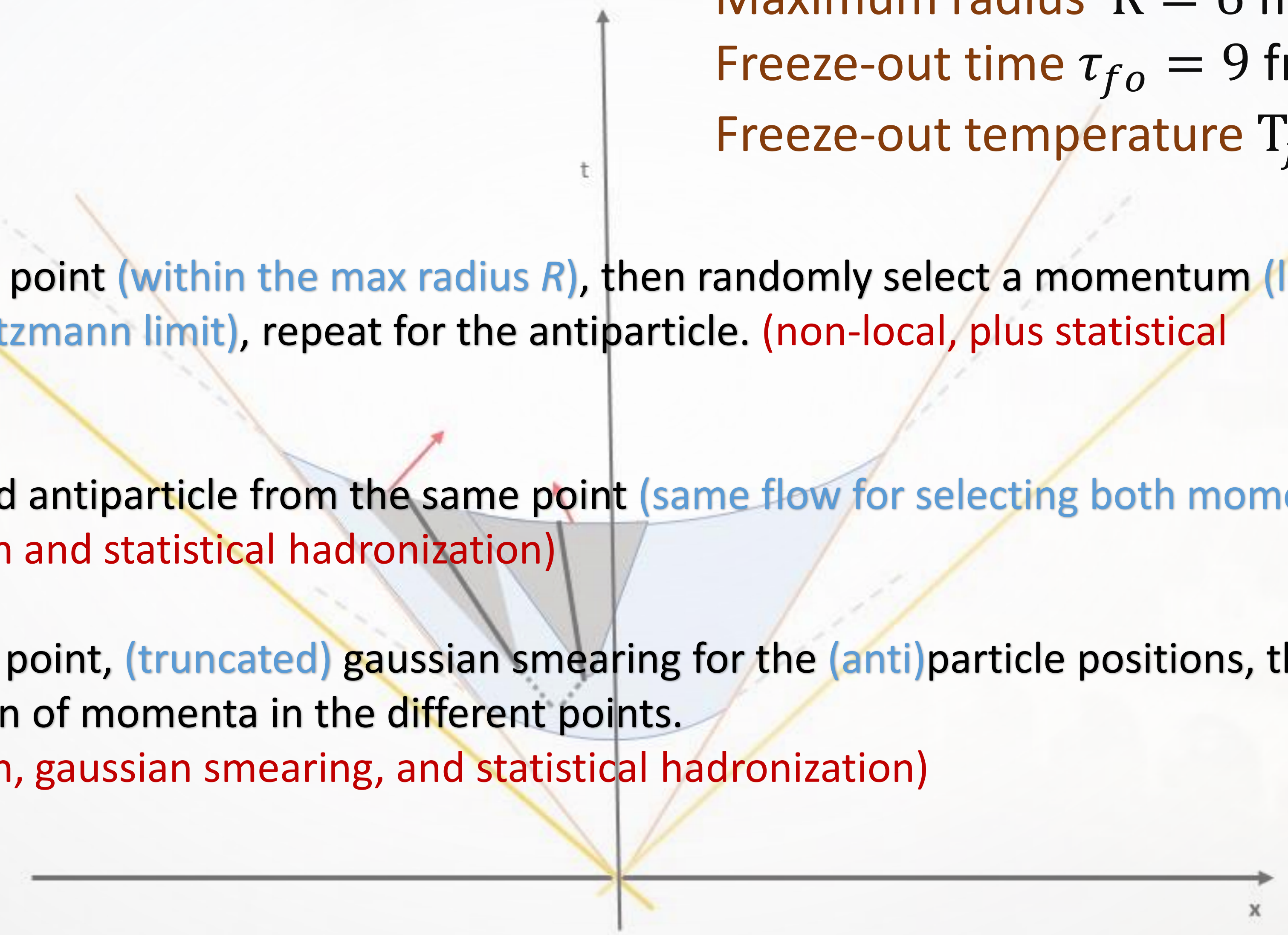
Freeze-out time $\tau_{fo} = 9$ fm/c

Freeze-out temperature $\mathrm{T}_{fo} = 150$ MeV

1. Select a random point (within the max radius *R*), then randomly select a momentum (local equilibrium, Boltzmann limit), repeat for the antiparticle. (non-local, plus statistical hadronization)

2. Both particle and antiparticle from the same point (same flow for selecting both momenta). (local production and statistical hadronization)

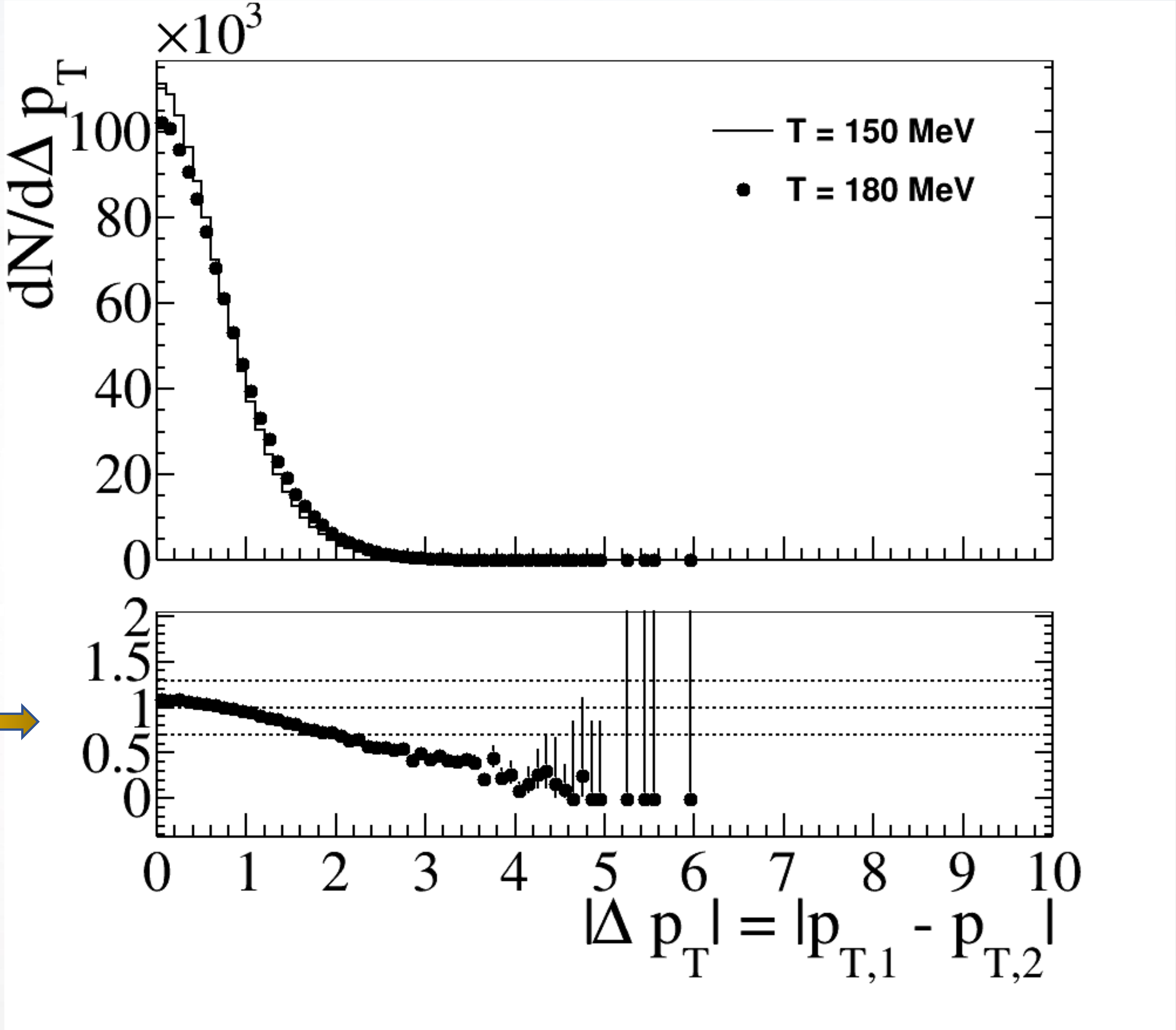
3. Select a starting point, (truncated) gaussian smearing for the (anti)particle positions, then random selection of momenta in the different points. (local production, gaussian smearing, and statistical hadronization)



Results

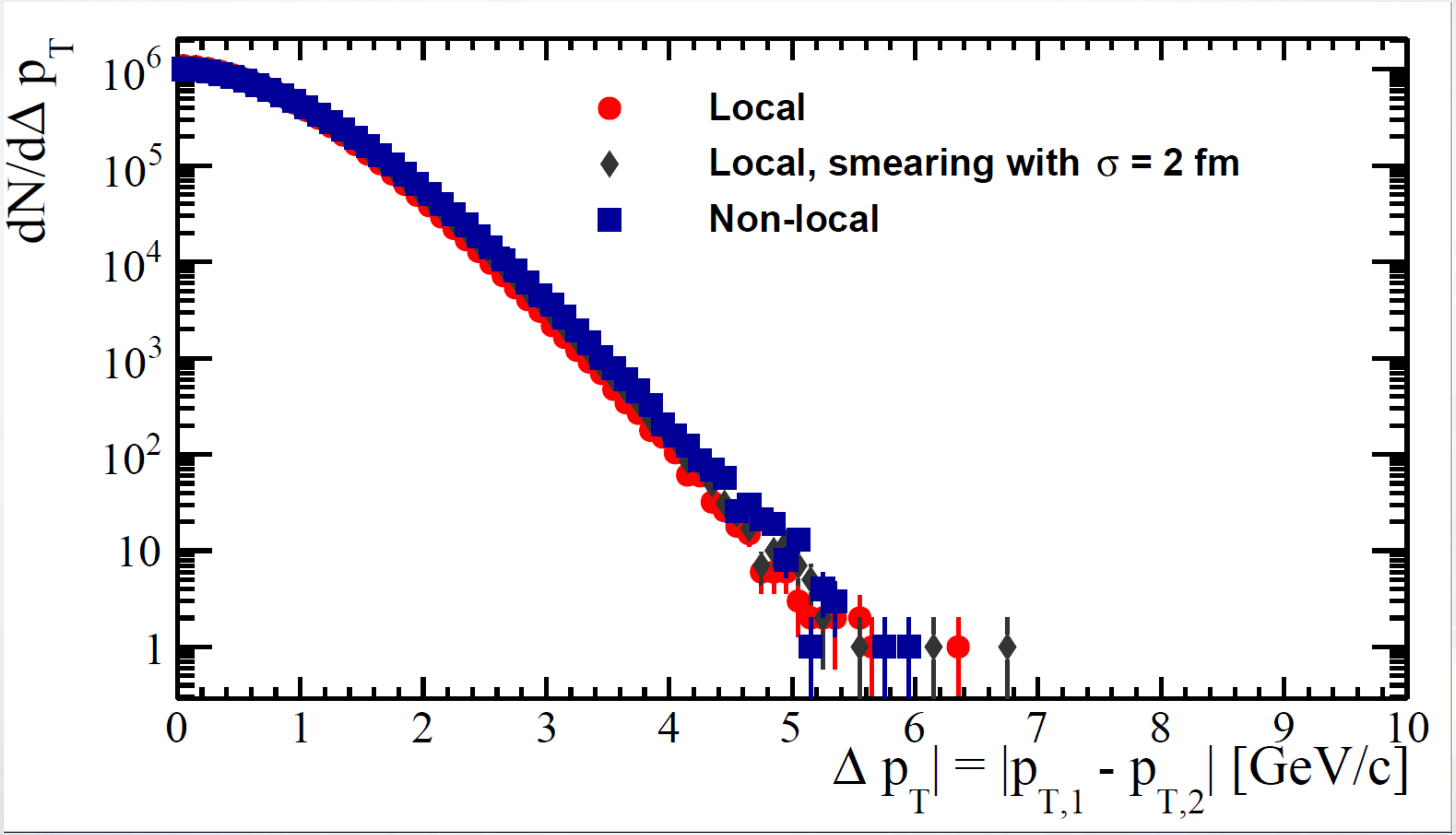
Transverse momenta (weakly) dependent on the freeze-out temperature.

Ratio of the counts between the two temperatures



Results

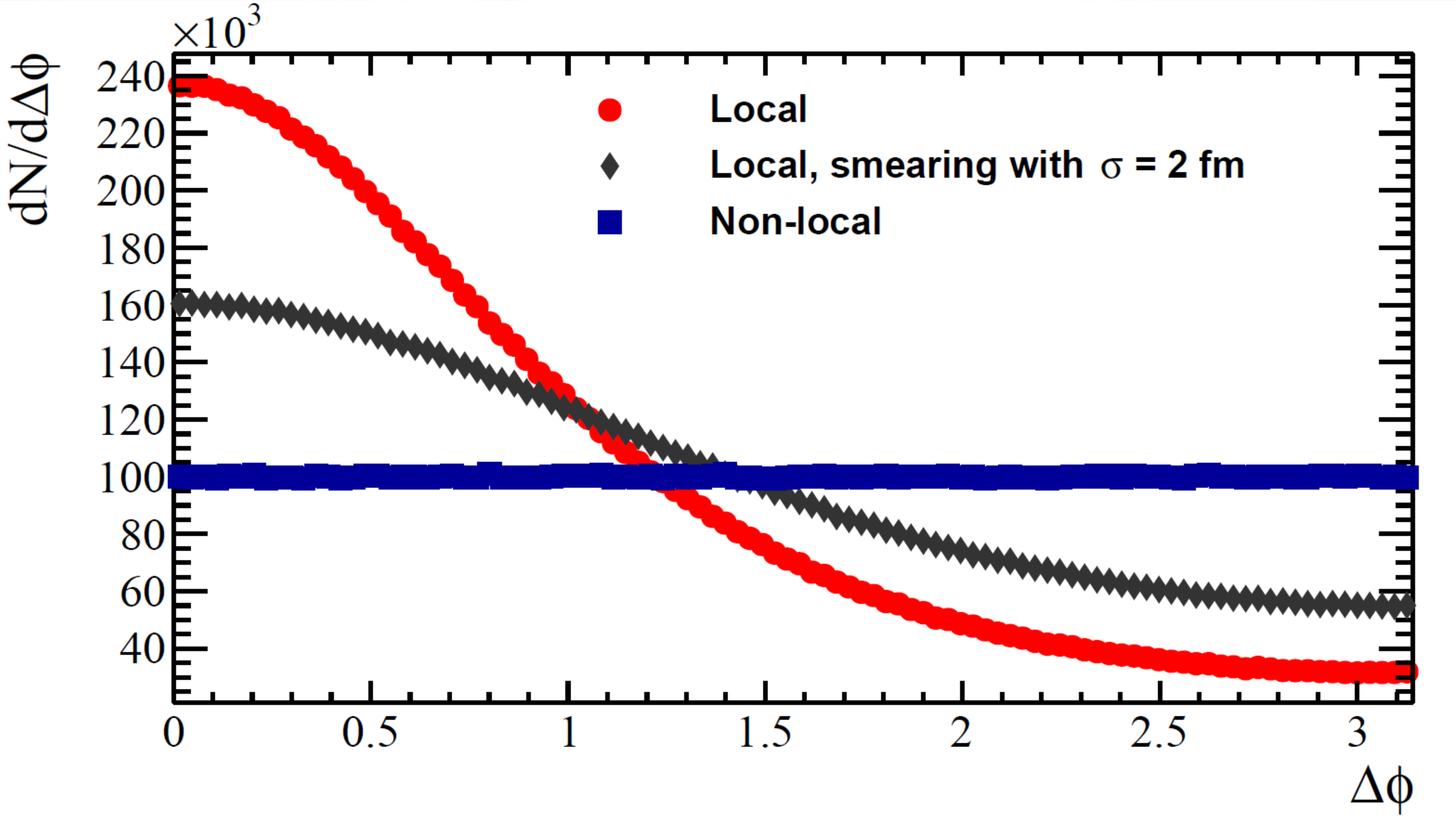
The thermal distribution almost "covers the correlations"



Results

Note: model dependence!

Larger diffusion flattens them



Results

How many events do we need to measure?

$$\langle D^0 \bar{D}^0 \rangle_{rec} = \langle c\bar{c} \rangle \cdot \left(P(c \to D^0) \cdot \mathrm{BR}(D^0 \to K\pi) \cdot P(\mathrm{acc.}) \cdot P(\mathrm{bkg.\ cuts}) \cdot P(\mathrm{rec.}) \right)^2$$

WIP

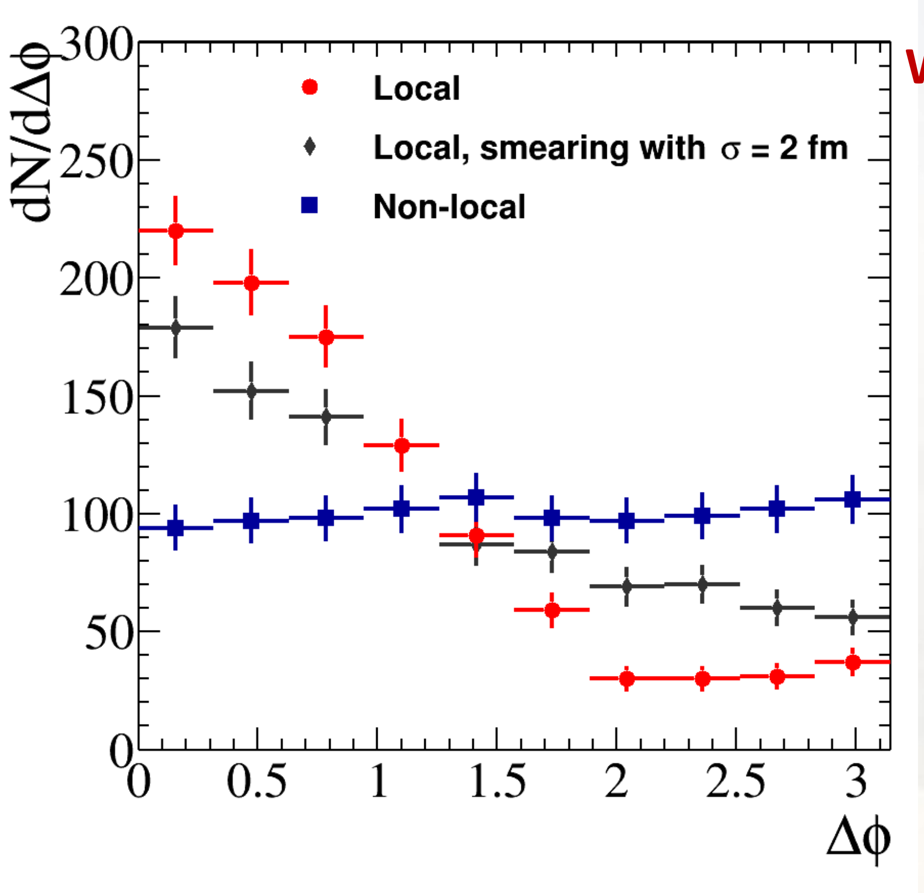
$P(c \to D^0)$	0.31	According to PHSD model [7].
$\mathrm{BR}(D^0 \to K^- \pi^+)$	0.398	According to PDG [8].
$P(\mathrm{acc.})$	0.5	The value was obtained from a Geant4 simulation with the nominal NA61/SHINE detector setup for Nov 2022.
$P(\mathrm{bkg.\ cuts})$	0.2	The value is taken from the recent analysis on D^0 and $\bar{D}^0$ mesons by A. Merzlaya [9].
$P(\mathrm{rec.})$	0.9	The value was obtained from a Geant4 simulation of the NA61/SHINE detector.

Results

How many events do we need to measure?

depending on the average number of $c - \bar{c}$ produced and read-out rate, for 1000 reconstructed meson pairs

	$\langle c\bar{c}\rangle = 0.1$	$\langle c\bar{c}\rangle = 0.2$	$\langle c\bar{c}\rangle = 0.5$	$\langle c\bar{c}\rangle = 1$
1 kHz	$\sim$ 1000 days	$\sim$ 500 days	$\sim$ 200 days	$\sim$ 100 days
10 kHz	$\sim$ 100 days	$\sim$ 50 days	$\sim$ 20 days	$\sim$ 10 days
100 kHz	$\sim$ 10 days	$\sim$ 5 days	$\sim$ 2 days	$\sim$ 1 day



WIP

Conclusions and outlook

- Incomplete models in the standard picture (evolution of the expectation values only)
- Open charm correlations to select the appropriate phenomenological extension
- Non-trivial implications physics wise

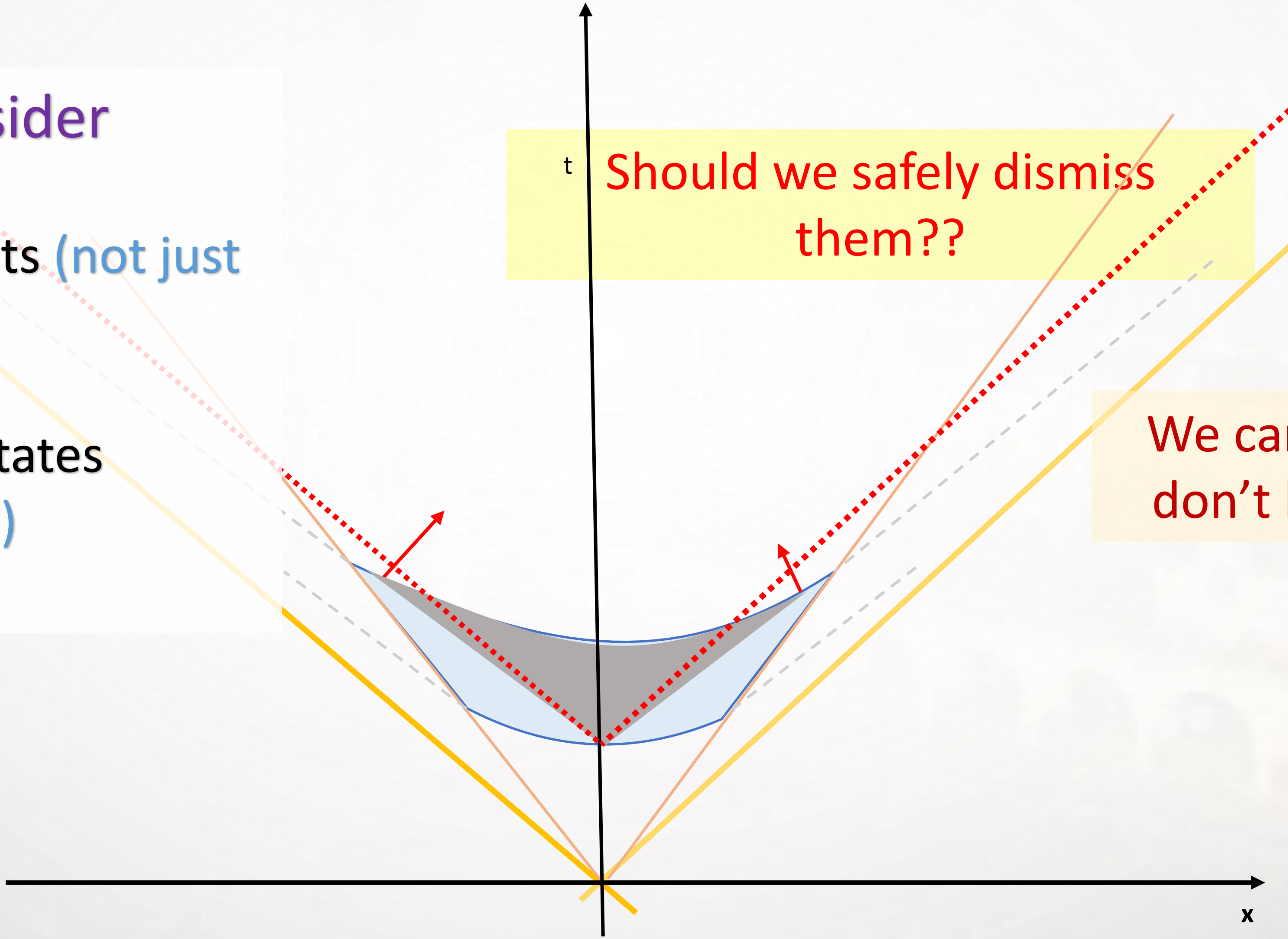
Back up slides

Intuitive *(but wrong)* assumptions

Some final positions of the charmed charges (depending on the geometry of the expansion) are not accessible without superluminal displacements

Details to consider

- Conserved currents (not just charges)
- Medium effects
- Structure of the states (and the measure)



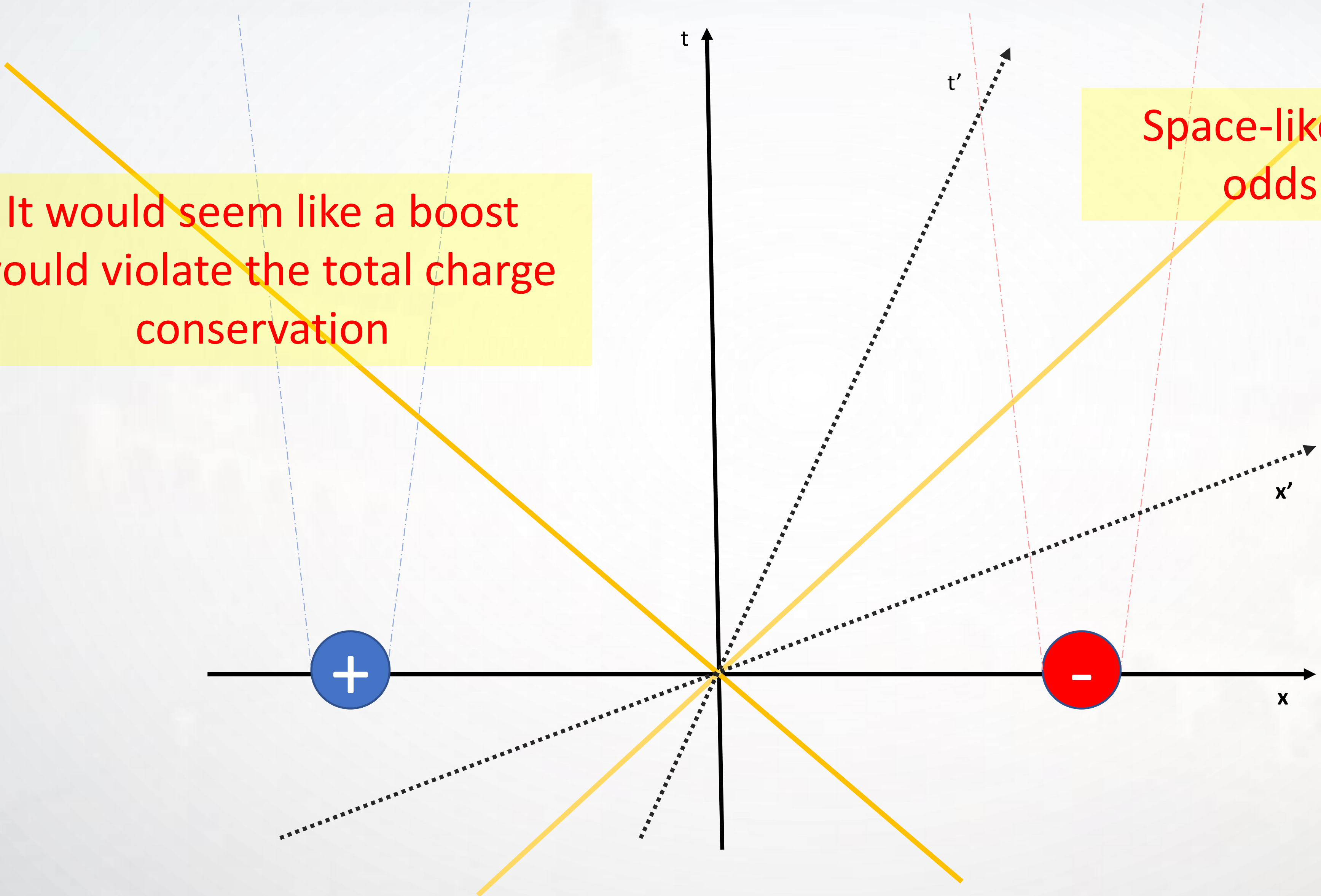
Should we safely dismiss them??

We can, but we don't have to…

Paradox: non-local production and teleportation of conserved charges

It would seem like a boost would violate the total charge conservation

Space-like "jumps" seem at odds with causality



x

Paradox: non-local production and teleportation of conserved charges

Can be solved considering the full four-current

Charge conservation preserved, boosted charge density taking an extra current-dependent term

$$\rho' = \gamma(\rho - \boldsymbol{v} \cdot \boldsymbol{J})$$

Classical analogue, antenna in electrodynamics