Time crystal phenomena in ultra-cold atoms bouncing on an oscillating mirror

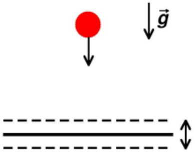
Arkadiusz Kuroś

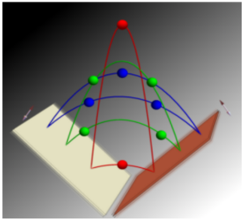
Institute of Physics, Jan Kochanowski University, Kielce

Outline

- Ultra-cold atoms bouncing on an oscillating mirror(s)
  - discrete time crystal and quasicrystal
  - crystalline structure in the time domain

- 1D
- 2D
- ND

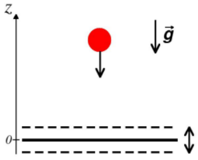




?

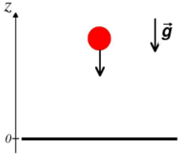
Single particle bouncing on the mirror

- periodically oscillating mirror

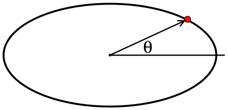


- $H = \frac{p^2}{2} + z + F(z + \lambda f(t))$
- $f(t) = f(t + \frac{2\pi}{\omega})$

- static mirror
  system is integrable $\rightarrow$ all trajectories are periodic orbit

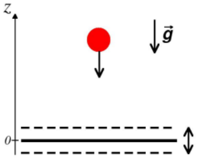


- $H_0 = \frac{1}{2}(3\pi I)^{3/2}$, $I = const$
- $\theta = \Omega t + const$, $\Omega = \frac{dH_0}{dI}$
  $\theta \in [0, 2\pi)$



Single particle bouncing resonantly on an oscillating mirror

resonant condition $\omega = n\Omega$ $\rightarrow$ only resonant trajectories are periodic orbits



- transformation to the frame moving along a resonant orbit $\Theta = \theta - \frac{\omega}{s} t$ resonant trajectories are stationary

- $H = \frac{p^2}{2} + z + F(z + \lambda f(t))$
- $H_0 = \frac{1}{2}(3\pi I)^{3/2}$, $I = const$
- $\theta = \Omega t + const$, $\Omega = \frac{dH_0}{dI}$ $\theta \in [0, 2\pi)$

- averaging over the fast time variable

- effective Hamiltonian that describes motion of a particle close to a resonant orbit

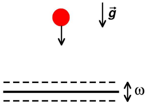
$$H_{eff} = \frac{P^2}{2m_{eff}} + \sum f_{-k} h_{ks} e^{iks\Theta}$$

Single particle bouncing on an oscillating mirror

example: $f(t) = \cos(\omega t)$ where $\omega = n\Omega$

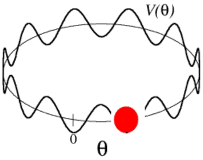
- laboratory frame

$$H = \frac{p^2}{2} + z + F(z + \lambda \cos(\omega t))$$

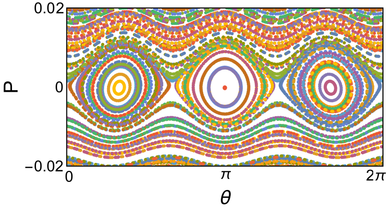


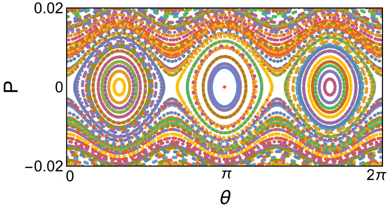
effective description

$$H_{eff} = \frac{P^2}{2m_{eff}} + V_0 \cos(n\Theta)$$



- phase space portraits $(n = 3)$ $\lambda \ll 1$





Single particle bouncing on an oscillating mirror

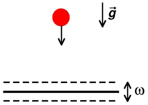
example: $f(t) = \cos(\omega t)$ where $\omega = n\Omega$

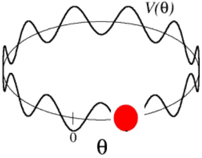
- laboratory frame

effective description

$$H = \frac{p^2}{2} + z + F(z + \lambda \cos(\omega t))$$

$$H_{eff} = \frac{P^2}{2m_{eff}} + V_0 \cos(n\Theta)$$





- crystalline structure in time domain

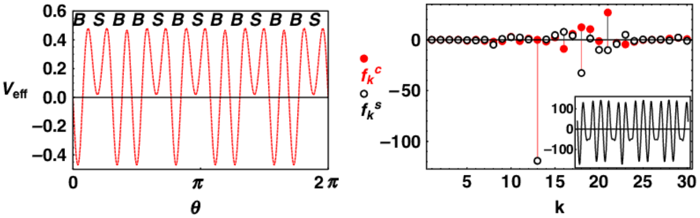
$$\Theta = \theta - \frac{\omega}{n} t$$

Quasicrystal structure in time domain

- periodic driving

$$f(t) = \sum f_k^c \cos(k\omega t) + f_k^s \sin(k\omega t)$$

- substitution rule $B \to BS$ and $S \to B$
  $B \to BS \to BSB \to BSBBS \to BSBBSBSB \to BSBBSBSBBSBBS$



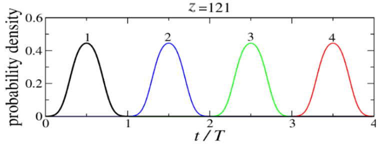
*K. Giergiel, A. Miroszewski and K. Sacha, PRL **120** 140401 (2018).*

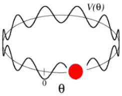
Oscillating mirror: quantum description

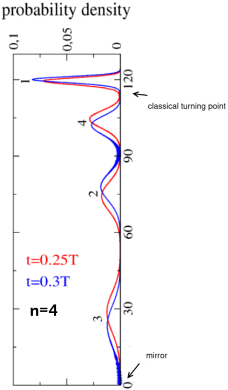
- time periodicity → Floquet Hamiltonian

$$\mathcal{H} = \frac{p^2}{2} + z + F(z + \lambda \cos(\omega t)) - i\partial_t$$

- n Floquet eigenstates $\phi_k(z, t+T) = \phi_k(z, t)$
- quasi-energies form a band when $n \to \infty$
- n individual wavepackets $W_j(z, t)$







- In the Wannier basis $\psi \approx \sum_i a_i W_i$

$$\mathcal{H}_{\text{eff}} \approx -\frac{1}{2} \sum_{\langle \mathbf{ij} \rangle} J_{\mathbf{ij}}\, a_{\mathbf{i}}^* a_{\mathbf{j}}$$

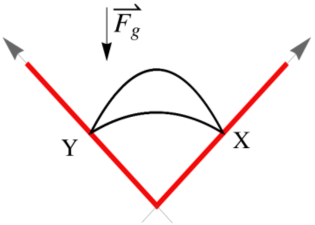
K.Sacha, Sci. Rep. 5, 10787 (2015).

Two-dimensional temporal lattices

Single particle bouncing between two orthogonal mirrors

static mirrors

- cartesian coordinates

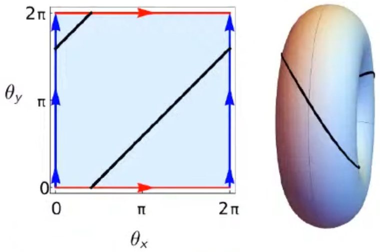


- the mirrors are located around $x = 0$ and $y = 0$
- the system is integrable

- action-angle variables

- $I_{x,y} = const$, $\theta_{x,y} = \Omega_{x,y} t + \theta_{x,y}(0)$

$\theta_x, \theta_y \in [0, 2\pi)$

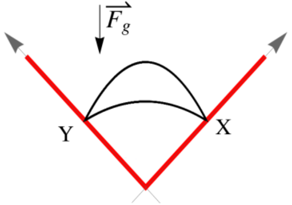


- torus boundary conditions

Single particle bouncing between two orthogonal mirrors

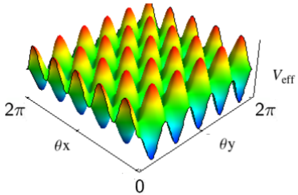
resonantly oscillating mirrors with frequency $\omega = n_{x,y}\Omega_{x,y}$

- laboratory frame



- the mirrors are located around $x = 0$ and $y = 0$
- only resonant trajectories are periodic

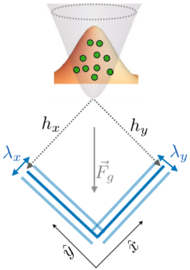
- effective description
- we are interested in motion of a particle close to a resonant orbit



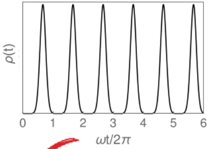
- effective potential describes $n_x \times n_y$ lattice
- various separable lattice geometries

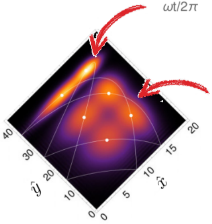
Two-dimensional temporal lattices

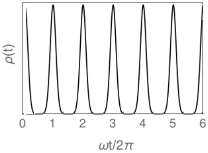
BEC with attractive interactions
in the lowest mode of a
2D harmonic trap



probabilities for the detection of atoms







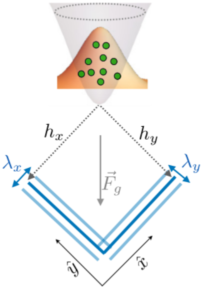
density of atoms bouncing
between two oscillating mirrors
$t = 2\pi/3\omega$

$$H(t) = \sum_{\alpha=x,y}\left[\frac{p_\alpha^2}{2} + \alpha + \lambda_\alpha \alpha \cos(\omega t + \delta_\alpha)\right], \quad \alpha \geqslant 0$$

Different phases in two-dimensional temporal lattices

- laboratory frame

BEC with attractive interactions
in the lowest mode of a
2D harmonic trap



$$H(t) = \sum_{\alpha=x,y}\left[\frac{p_\alpha^2}{2} + \alpha + \lambda_\alpha \alpha \cos(\omega t + \delta_\alpha)\right], \quad \alpha \geqslant 0$$

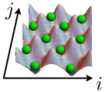
$$U_{\mathrm{ij}} = \frac{N}{T}\int_0^T g(t)dt \int dxdy\, |W_{\mathrm{i}}|^2 |W_{\mathrm{j}}|^2$$

- effective description

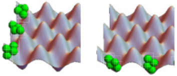
$$\hat{\mathcal{H}} \approx -\frac{1}{2}\sum_{\langle \mathrm{i,j}\rangle} J_{\mathrm{ij}}\, \hat{a}_{\mathrm{i}}^\dagger \hat{a}_{\mathrm{j}} + \frac{1}{2}\sum_{\mathrm{i,j}} U_{\mathrm{ij}}\, \hat{a}_{\mathrm{i}}^\dagger \hat{a}_{\mathrm{j}}^\dagger \hat{a}_{\mathrm{j}} \hat{a}_{\mathrm{i}}$$

Types of Phases

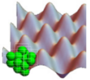
a) Symmetry preserving regime



b) Partial Symmetry breaking regime



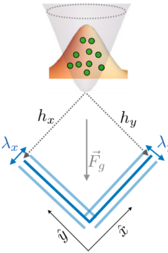
c) Complete Symmetry breaking regime



Different phases in two-dimensional temporal lattices

- laboratory frame

BEC with attractive interactions in the lowest mode of a 2D harmonic trap



$$H(t) = \sum_{\alpha=x,y} \left[\frac{p_\alpha^2}{2} + \alpha + \lambda_\alpha \alpha \cos(\omega t + \delta_\alpha) \right], \quad \alpha \geqslant 0$$

$$U_{ij} = \frac{N}{T} \int_0^T g(t) dt \int dxdy\, |W_i|^2 |W_j|^2$$

- effective description

$$\hat{\mathcal{H}} \approx -\frac{1}{2} \sum_{\langle i,j \rangle} J_{ij}\, \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} \sum_{i,j} U_{ij}\, \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_i$$

Types of Phases

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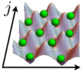

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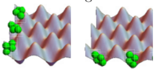

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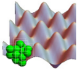
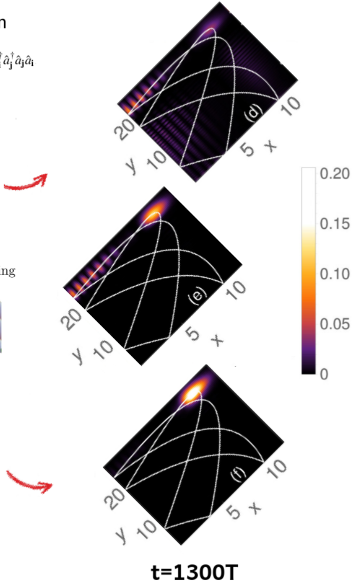


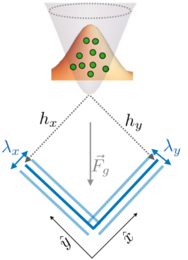


AK, R. Mukherjee, F. Mintert, K. Sacha, Phys. Rev. Res. 3 043203 (2021).

Different phases in two-dimensional temporal lattices

- laboratory frame

BEC with attractive interactions in the lowest mode of a 2D harmonic trap



$$H(t) = \sum_{\alpha=x,y}\left[\frac{p_\alpha^2}{2} + \alpha + \lambda_\alpha \alpha \cos(\omega t + \delta_\alpha)\right], \quad \alpha \geqslant 0$$

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$$\hat{\mathcal{H}} \approx -\frac{1}{2}\sum_{\langle \mathbf{i},\mathbf{j}\rangle} J_{\mathbf{ij}}\, \hat{a}_{\mathbf{i}}^\dagger \hat{a}_{\mathbf{j}} + \frac{1}{2}\sum_{\mathbf{i},\mathbf{j}} U_{\mathbf{ij}}\, \hat{a}_{\mathbf{i}}^\dagger \hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{j}} \hat{a}_{\mathbf{i}}$$

Types of Phases

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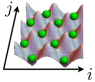

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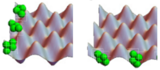

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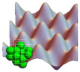
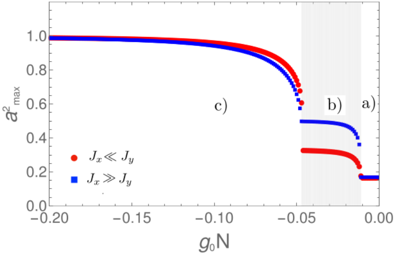




$$\psi_{gs}(x,y,t) \approx \sum_{\mathbf{i}} a_{\mathbf{i}} W_{\mathbf{i}}(x,y,t)$$

$$\sum_i a_i^2 = 1$$

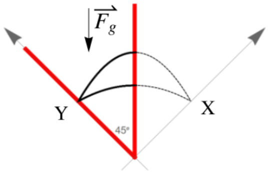
AK, R. Mukherjee, F. Mintert, K. Sacha, Phys. Rev. Res. 3 043203.

Implementation of non-separable 2D lattice in the time domain

Single particle bouncing in the wedge with the angle 45°

static mirrors

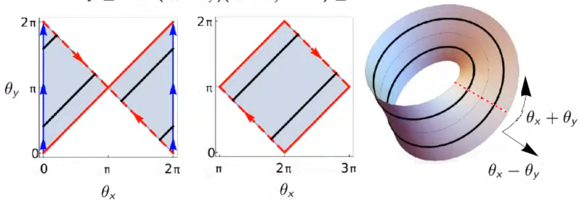
- cartesian coordinates



- the mirrors are located around x = 0 and x−y = 0
- collision with the vertical mirror
  $p_x \leftrightarrows p_y$

- action-angle variable of the orthogonal mirrors problem
- the system is integrable
  - local mapping to the orthogonal mirrors problem
- restricted domain

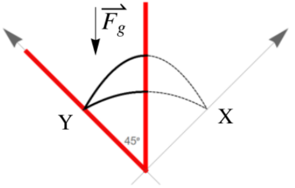
$$y \geq x \Leftrightarrow (\theta_x - \theta_y)(\theta_x + \theta_y - 2\pi) \geq 0$$



- Möbius strip boundary conditions

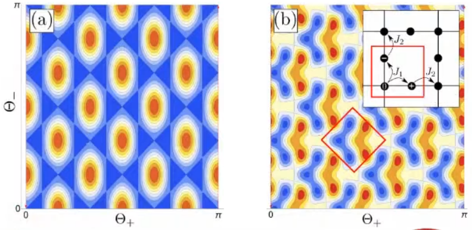
Non-separable 2D lattice in the time domain

- cartesian coordinates



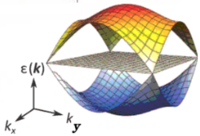
- the mirrors are located around $x = 0$ and $x - y = 0$
- collision with the vertical mirror
  $p_x \leftrightarrows p_y$

- effective description
  - if both mirrors oscilate – inseparable problem!
  - flexibility in designing lattice geometries.
    e.g. honeycomb, the Lieb lattice



The Lieb lattice:

- Bravais lattice with the three point basis.
- three energy bands (one flat band)



shaking protocols of two mirrors → lattice geometries

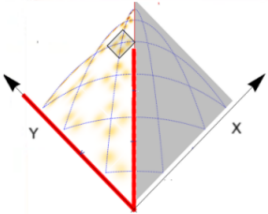
Ultra-cold bosonic atoms: 2D Lieb lattice

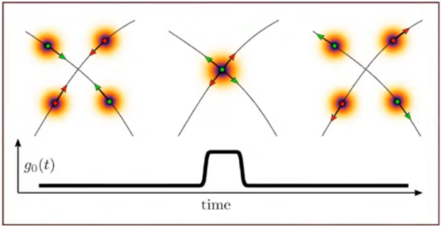
Why are we interested in a flat band?

- no dynamics in a non-interacting system
- interaction induced dynamics $\longrightarrow$ quantum simulator of exotic many-body physics
- many-body Floquet Hamiltonian restricted to the flat band subspace

$$H_F = \sum_{ijkl} U_{ijkl} \hat{b}_i^\dagger \hat{b}_j^\dagger \hat{b}_k \hat{b}_l + \text{const} \quad \text{with} \quad U_{ijkl} \propto \int \mathrm{d}t\, \mathrm{d}x\, \mathrm{d}y\; g_0(t)\; w_i^* w_j^* w_k w_l$$

- **Lab frame**: the Wannier states evolve periodically in the real space.
- Contact interactions, can be modulated in time $g_0(t) = g_0(t + nT)$

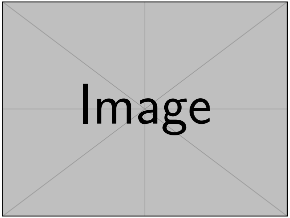


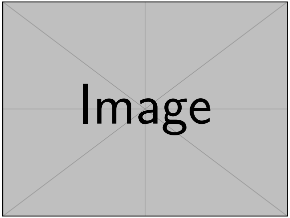


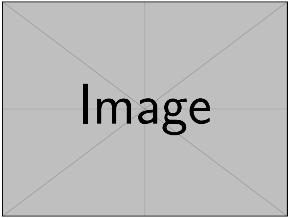
- Long-ranged interactions U_{ijkl} are tunable – Feshbach resonance!

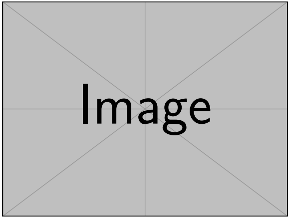
K. Giergiel, AK, A. Kosior, K. Sacha, Phys. Rev. Lett. 127, 263003 (2021)

N-dimensional temporal lattices









W. Golletz, A. Czarnecki, K. Sacha, AK, New Journal of Physics, 24, 093002 (2022).

Summary

- Unique platform for modelling crystalline structure in the time domain.
- Great flexibility in designing temporal N-dimensional lattice geometries.
- Comfortable platform for quantum simulations of flat band many-body physics.

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- Weronika Golletz, Jagiellonian University Kraków
- Krzysztof Giergiel, Swinburne University of Technology Melbourne
- Krzysztof Sacha, Jagiellonian University Kraków
- Andrzej Czarnecki, Jagiellonian University Kraków
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- Florian Mintert, Imperial College London

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