Exploring the QCD phase diagram with heavy-ion collisions

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Outline

- introduction

- theory vs. experiment

- cumulants, factorial cumulants, factorial moments

- expectations

- measurements and interpretation

- summary

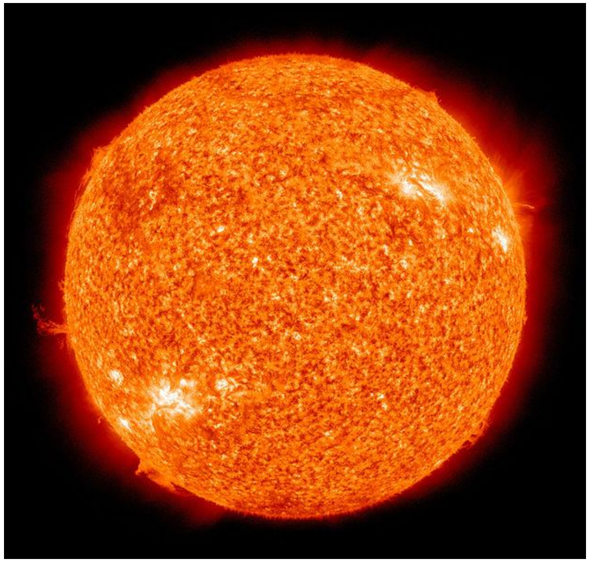
Quark-gluon plasma as a new state of matter

$T \sim 300$ K

$T \sim 1500$ K

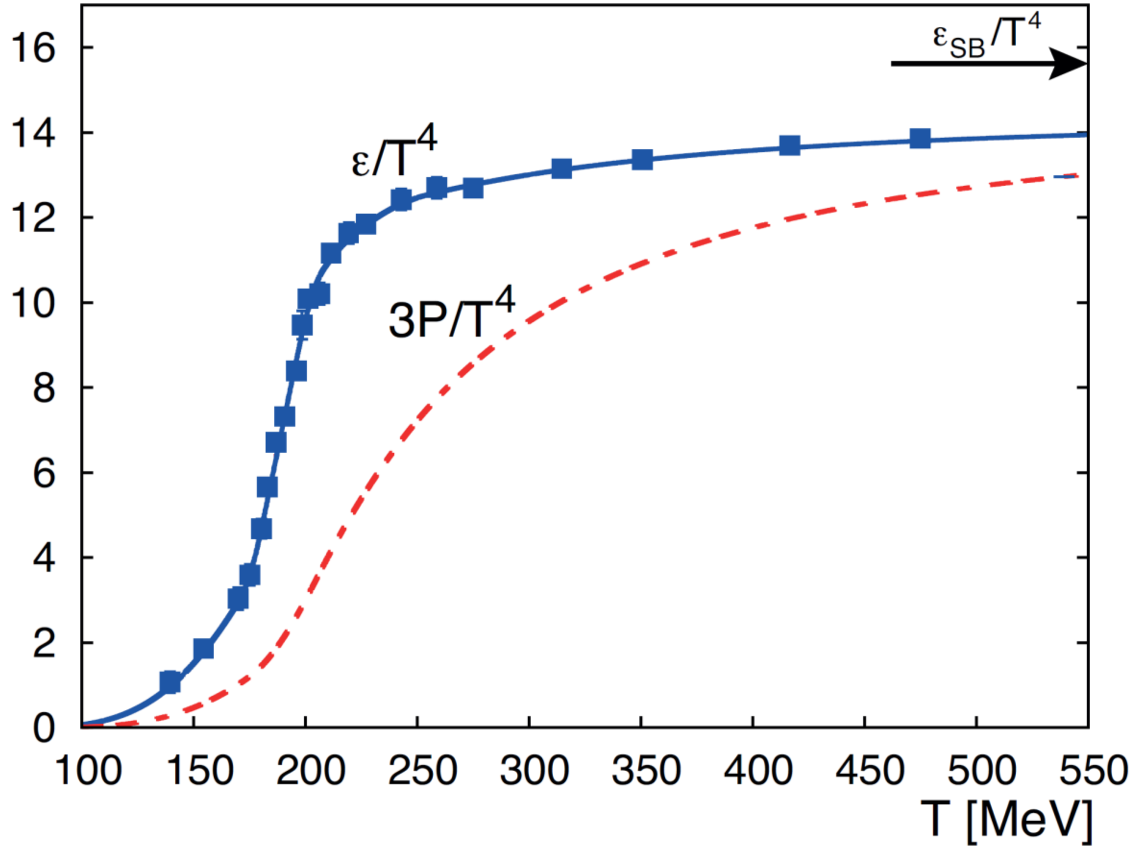
Gas of gold atoms: $T \sim 3000$ K

To melt protons: $T \sim 10^{12}$ K



Center of the Sun: $T \sim 10^{7}$ K

Lattice QCD calculations (LQCD)

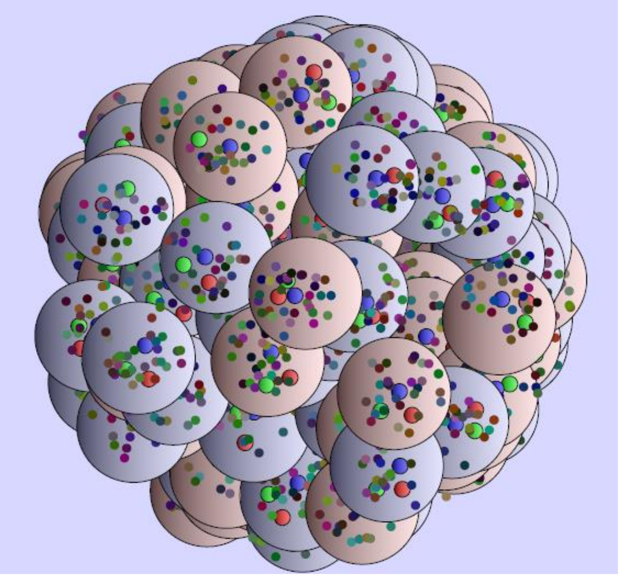


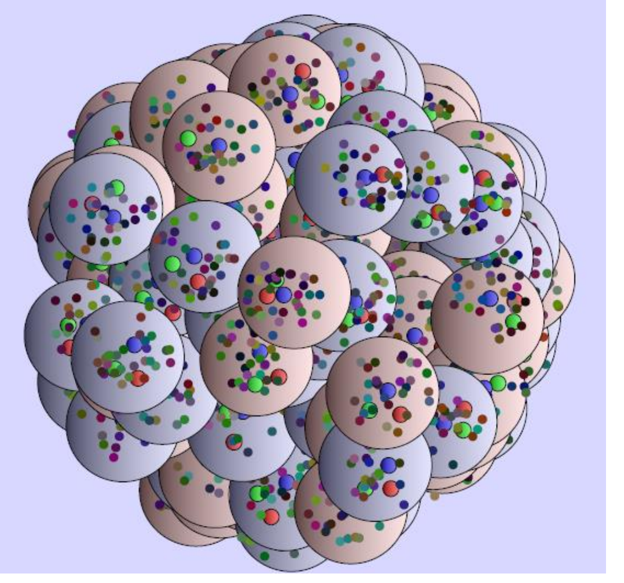
A smooth crossover

$$170 \text{ MeV} \approx 2 * 10^{12} \text{ K}$$

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature 443 (2006) 675
A.Bazavov, T.Bhattacharya, M.Cheng et al., Phys. Rev. D80, 014504 (2009)

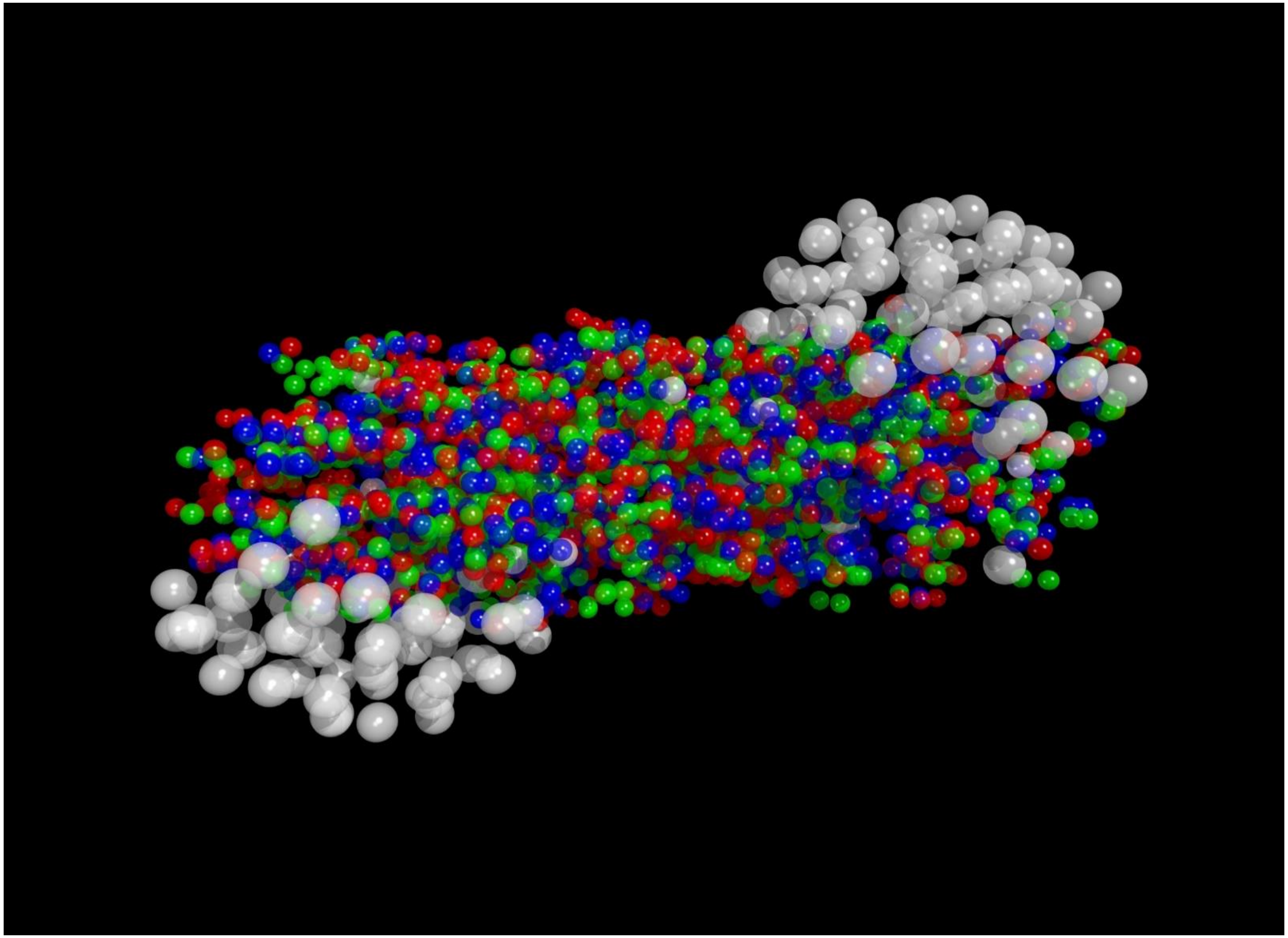
How to create and measure such temperature?





LHC – Large Hadron Collider
RHIC – Relativistic Heavy Ion Collider, Nowy Jork, USA
GSI Helmholtz Centre for Heavy Ion Research, Darmstadt, Niemcy

The collision creates a “quark-gluon plasma”



The QCD phase diagram

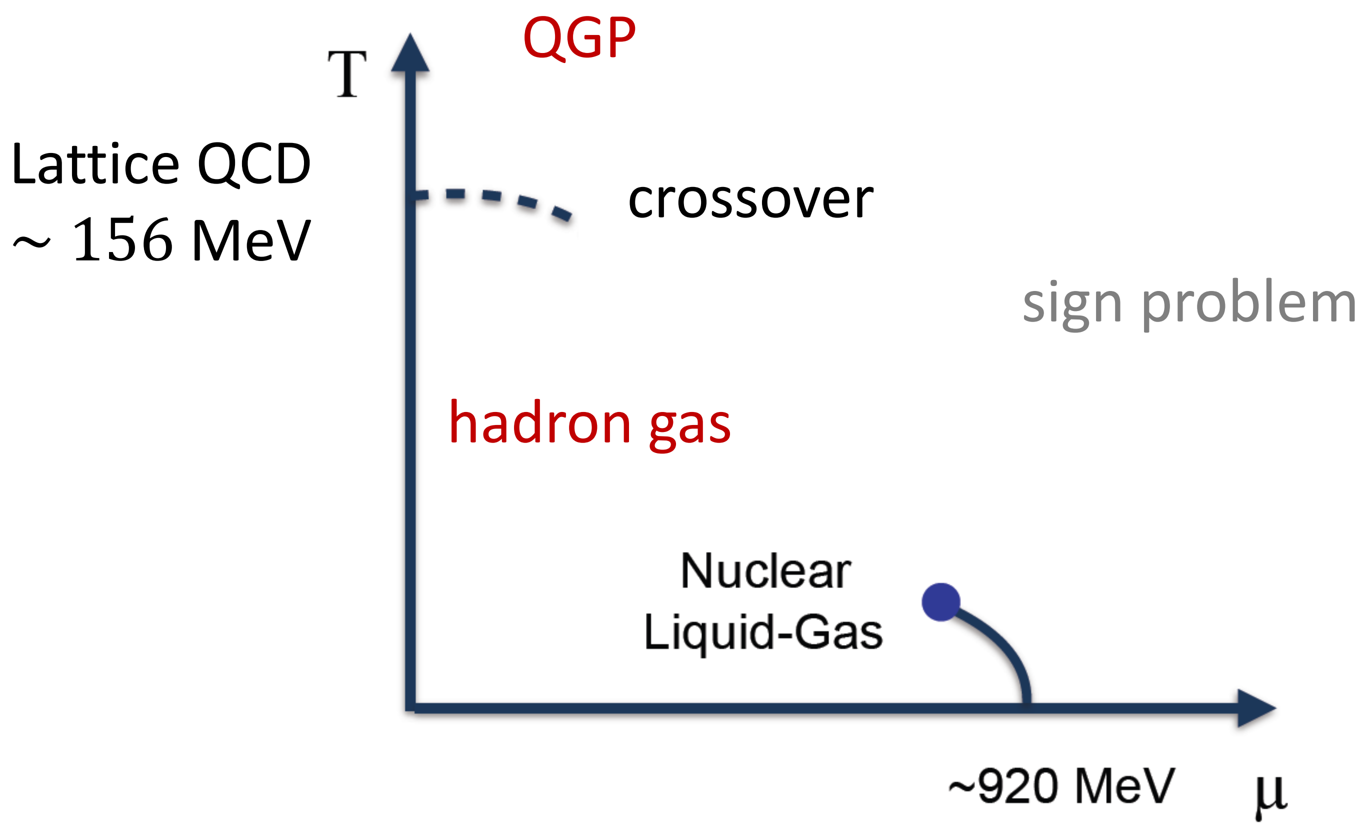


Figure from V.Koch

Expectations

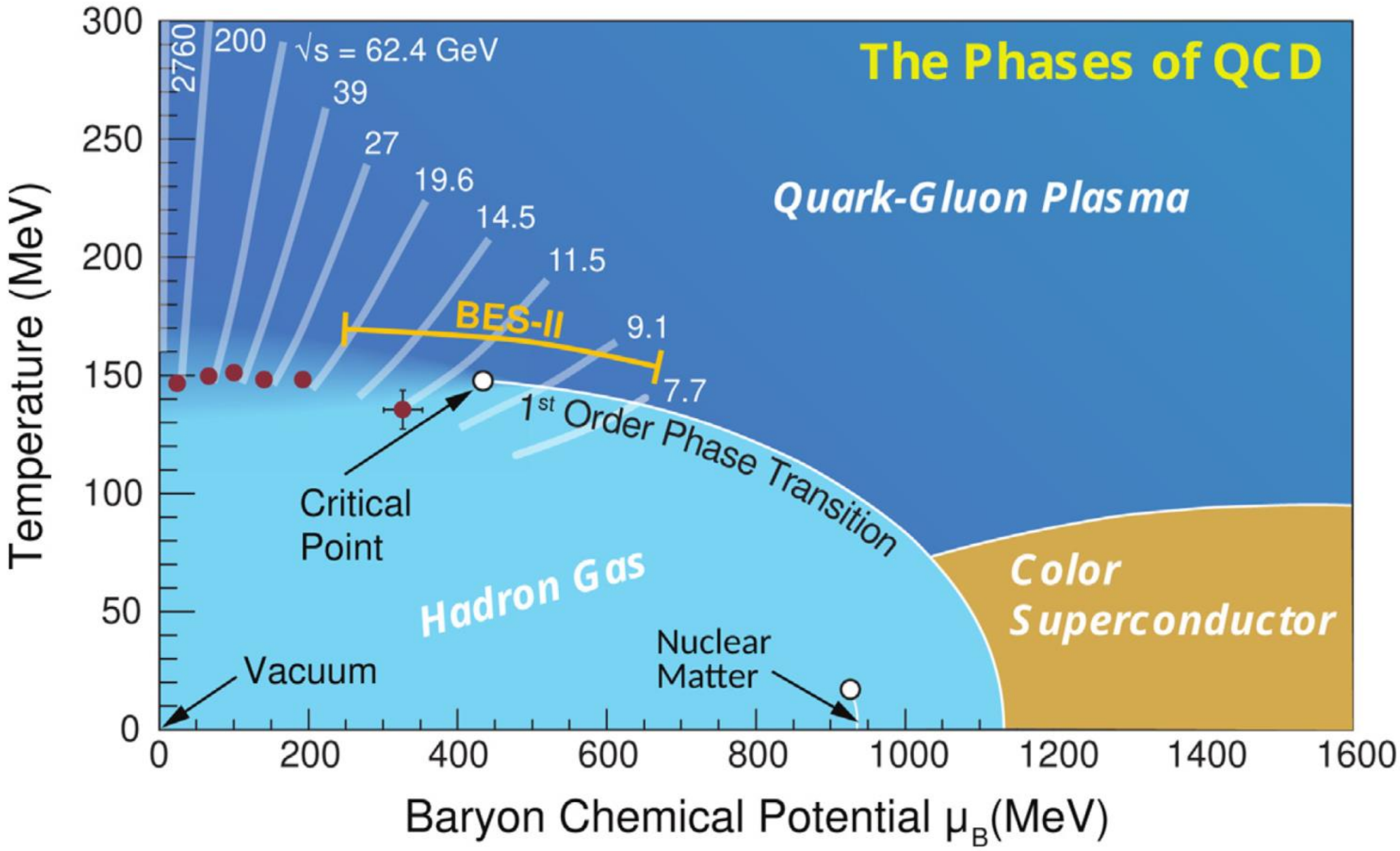
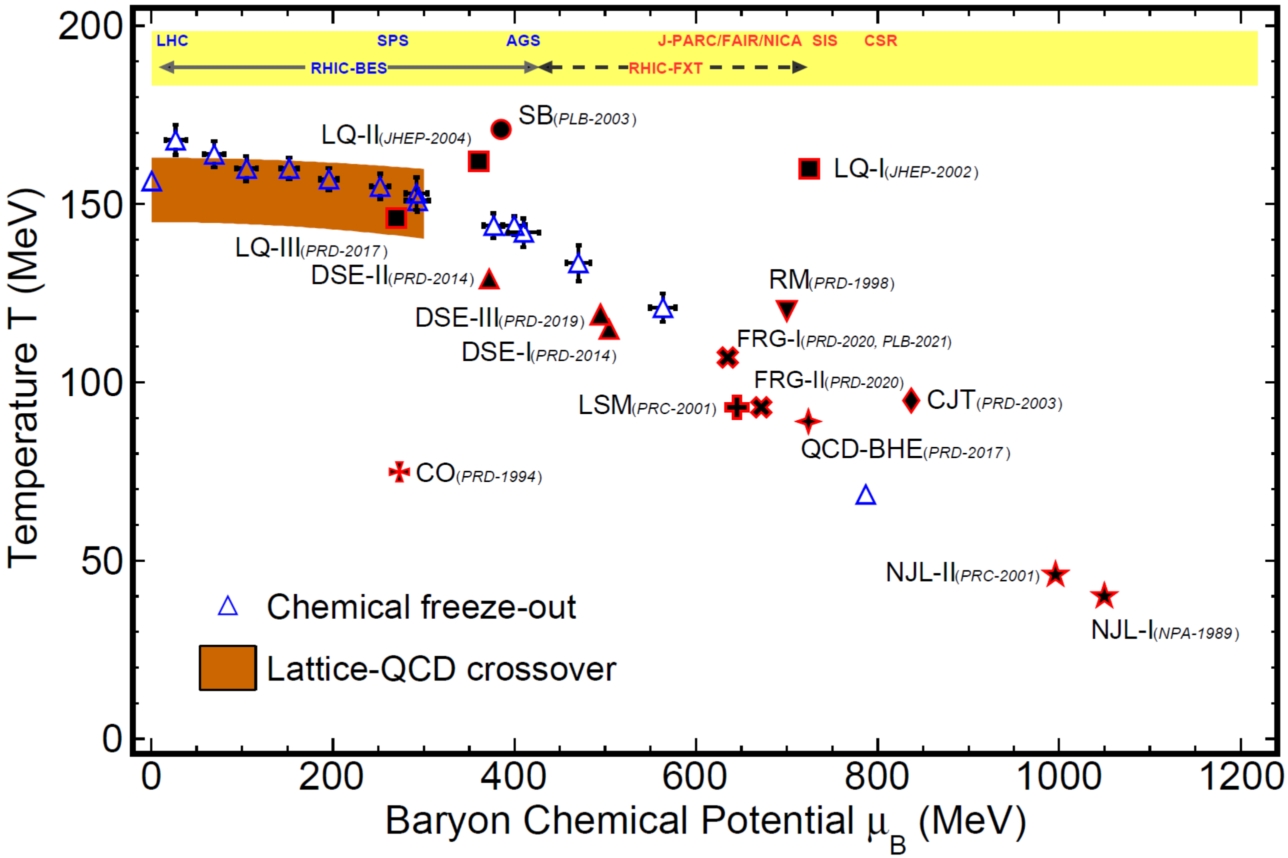


Figure from Phys. Rept. 853 (2020) (AB, S.Esumi, V.Koch, J.Liao, M.Stephanov, N.Xu)

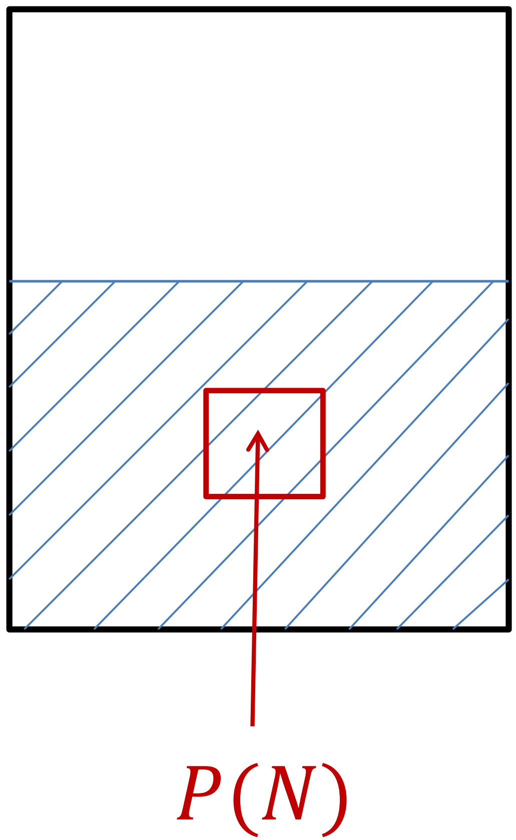
Critical point?



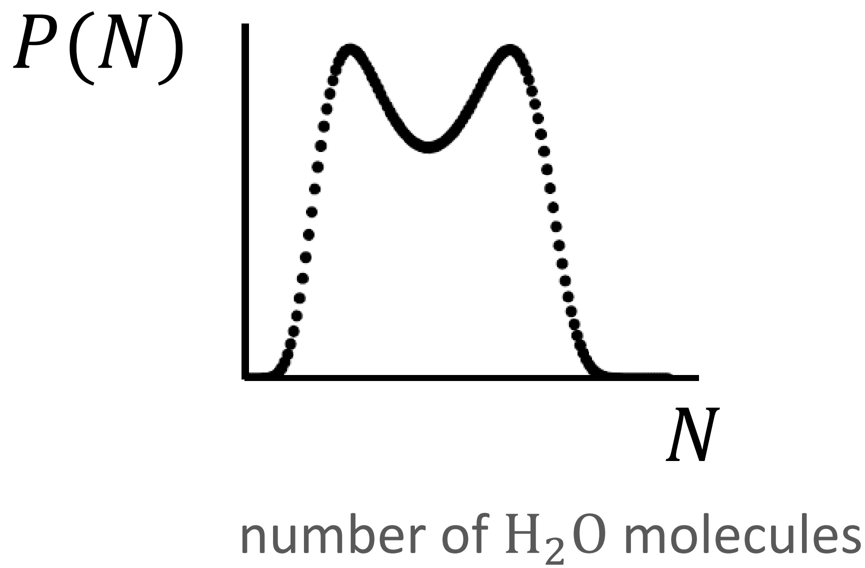
A. Pandav, D. Mallick, B. Mohanty, 2203.07817

M. Stephanov, hep-lat/0701002

How to approach this problem?
Consider water vapour transition



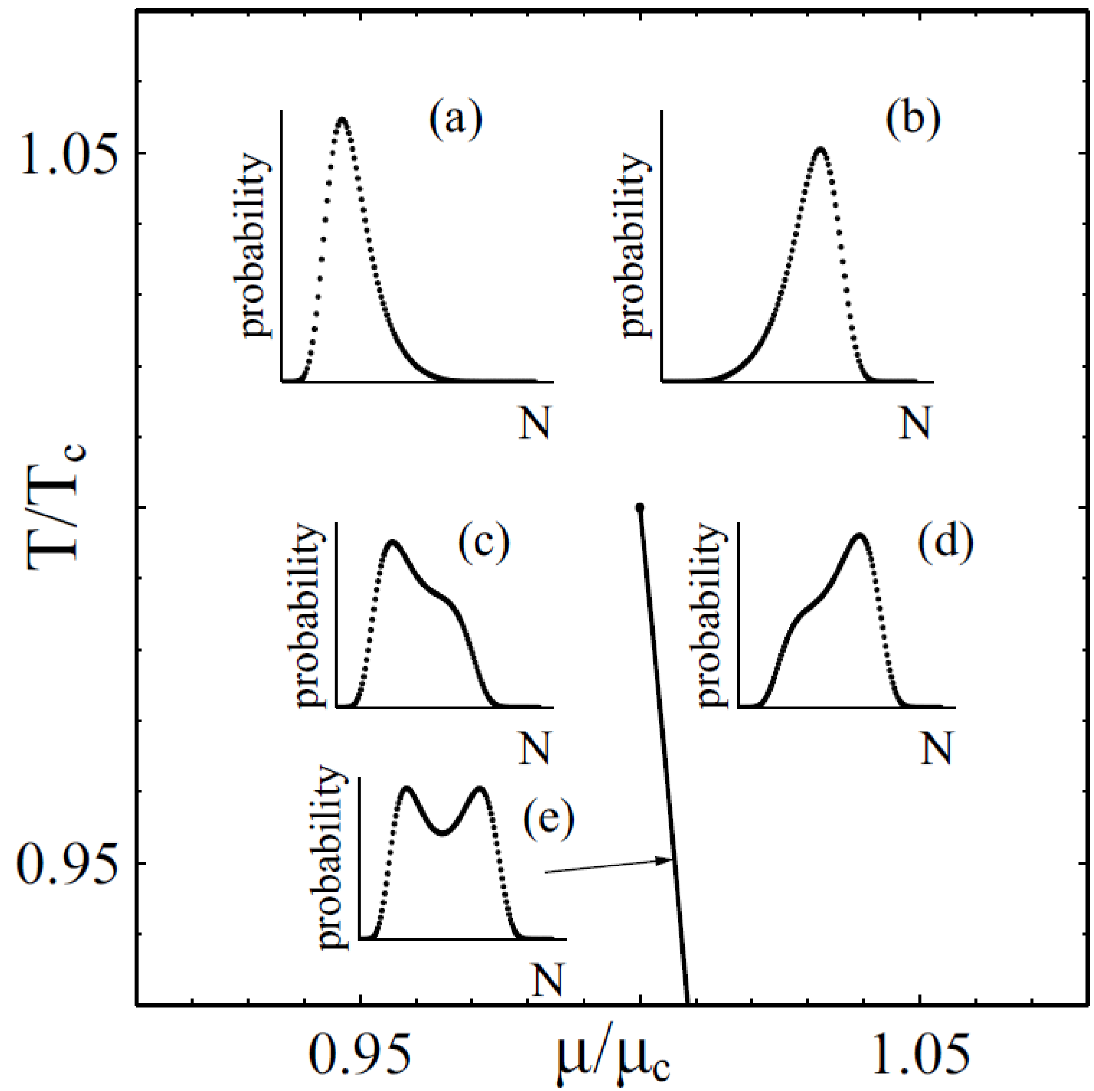
right at the phase transition



so we measure multiplicity distributions

In QCD we use, e.g., net-baryon, net-charge, net-strangeness

A finite volume van der Waals model



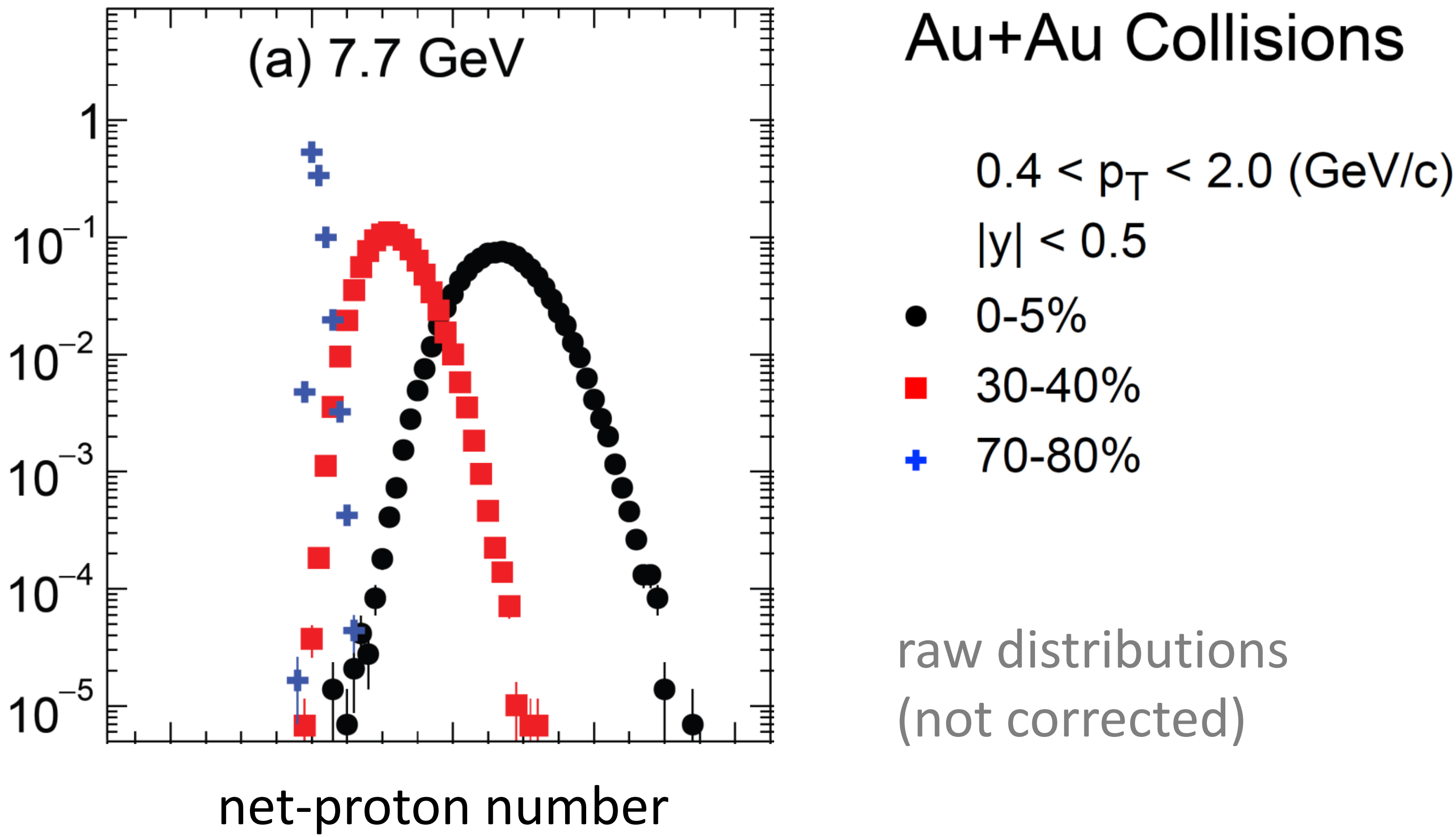
AB, V. Koch, D. Oliinychenko, J. Steinheimer, PRC 98 (2018) 054901

Theory vs. experiment

Theory	**Experiment**
Coordinate space	Momentum space
Fixed volume	Expanding and fluctuating volume
Long-lived	Extremely short-lived
Conserved charges	Non-conserved numbers

Lots of detector problems + various corrections (e.g. volume fluctuation)

So we measure multiplicity distributions



raw distributions
(not corrected)

STAR Collaboration, PRC 104 (2021) 2, 024902

For baryons absolutely minimal goal is to see any deviations from Poisson (Skellam) distribution.

It is difficult to see something in multiplicity distributions. If there is any signal, it is likely very tiny.

We usually characterize $P(N)$ by:

- cumulants κ_n

- factorial cumualnts, C_n (or $\hat{C}_n$)

- factorial moments F_n (mean number of pairs, triplets, etc.)

Warning. STAR uses opposite notation $\kappa_n \leftrightarrow C_n$

On the experimental side we need to measure various fluctuation observables and hope to see some nontrivial energy or/and system-size dependence

see, e.g.,
Stephanov, Rajagopal, Shuryak, PRL (1998)
Stephanov, PRL (2009)
Skokov, Friman, Redlich, PRC (2011)

There are many results:

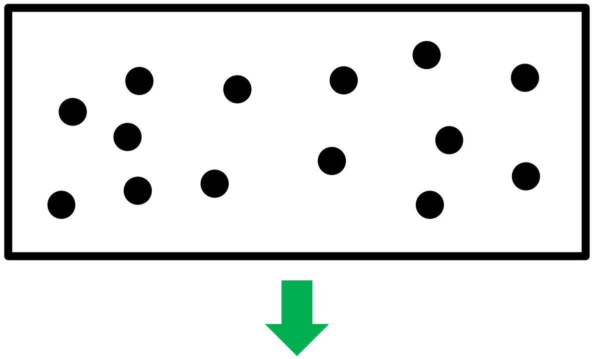
ALICE, STAR, HADES	NA61/SHINE
Cumulants, factorial cumulamts	Intermittency, cumulamnts
Proton v_1 (STAR)	Scaled variance
HBT radii (STAR) R.A. Lacey, PRL 114 (2015) 142301	Strongly intensive variables

Poisson distribution (no correlations)

$N = 10^{10}$

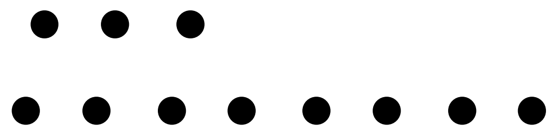
$p = 10^{-9}$

$\langle n \rangle = Np = 10$



event # 1

event # 2



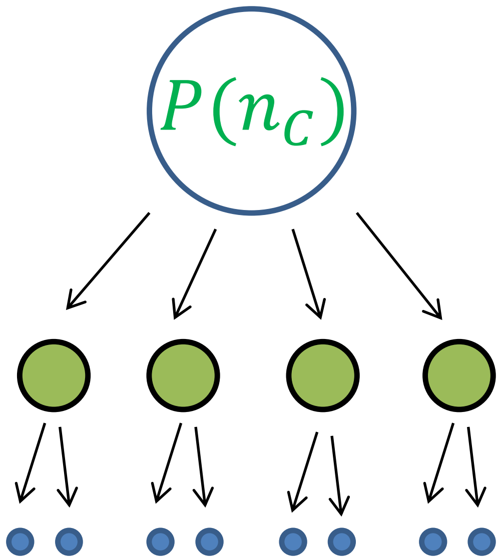
$P(n) =$ Poisson if $N \to \infty,\ p \to 0,\ Np = \langle n \rangle$

cumulants $\kappa_i = \langle n \rangle$

factorial cumulants $C_i = 0$

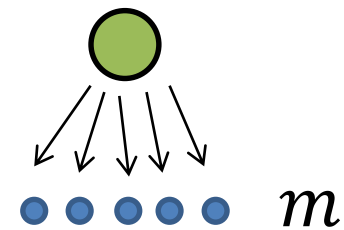
factorial moments $F_i = \langle n \rangle^i$

Factorial cumulants – example



Poisson

m particle cluster



$C_2 \neq 0$

$C_k = 0, k > 2$

$C_{2,3,\ldots,m} \neq 0$

$C_k = 0, k > m$

factorial cumulants

$$C_k = \frac{d^k}{dz^k} \ln\left(\sum_n P(n) z^n\right)\Big|_{z=1}$$

Two-particle correlation function

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + C_2(y_1, y_2)$$

Integrating both sides over some bin in rapidity

$$\langle n(n-1)\rangle = \langle n\rangle^2 + C_2$$

$$C_2 = \int C_2(y_1, y_2)dy_1 dy_2$$

Same with multiparticle correlations.

Factorial cumulants are integrated multiparticle correlation functions

Factorial cumulants vs cumulants

factorial cumulant

$$C_i = \frac{d^i}{dz^i} \ln\left(\sum_n P(n) z^n\right) |_{z=1}$$

cumulant

$$\kappa_i = \frac{d^i}{dt^i} \ln\left(\sum_n P(n) e^{tn}\right) |_{t=0}$$

Poisson

$C_i = 0,\ \kappa_i = \langle n \rangle$

cumulants naturally appear in statistical physics

$$\ln(Z) = \ln\left(\sum_i e^{-\beta(E_i - \mu N_i)}\right)$$

Cumulants (one species of particles)

$$\kappa_2 = \langle N \rangle + C_2$$

$$\kappa_3 = \langle N \rangle + 3C_2 + C_3$$

$$\kappa_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

Cumulants mix integrated correlation functions of different orders

They might be dominated by $\langle N \rangle$.

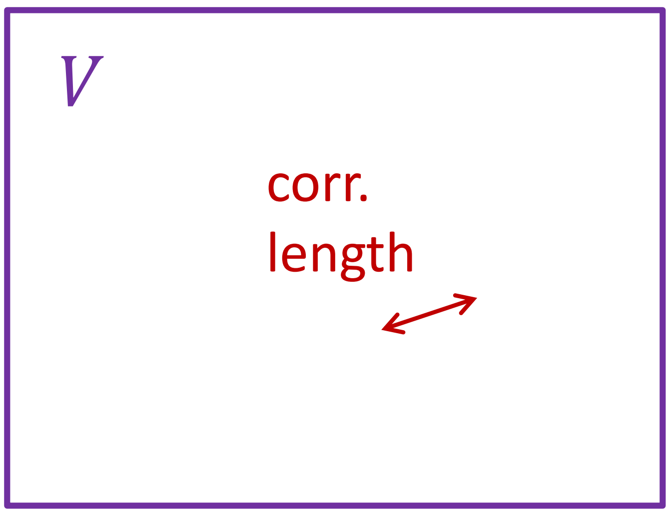
See, e.g.,
B. Ling, M. Stephanov, PRC 93 (2016) 034915
AB, V.Koch, N.Strodthoff , PRC 95 (2017) 054906

“Cumulant ratios do not depend on volume”

but depend on volume fluctuation

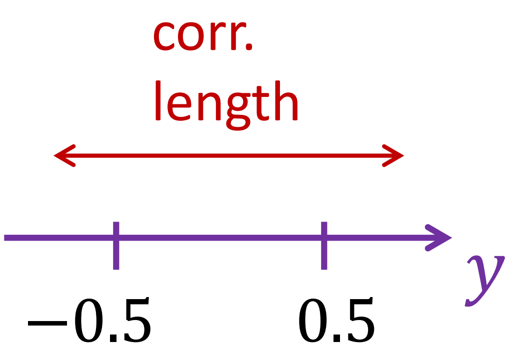
It is true if a correlation length is much smaller than the system size

coordinate space



Here this condition is satisfied

momentum rapidity space



Correlation length is usually larger than one unit of rapidity.

Cumulant ratios are expected to depend on acceptance in rapidity

Short-range correlations

$$C_i \sim \langle N \rangle \sim \Delta y$$

$$\kappa_i \sim \langle N \rangle \sim \Delta y$$

Long-range correlations (expected in rapidity)

$$C_i \sim \langle N \rangle^i \sim (\Delta y)^i$$

κ_i is complicated, for example

$$\kappa_4 = \langle N \rangle + \left(\sim\langle N \rangle^2\right) + \left(\sim\langle N \rangle^3\right) + \left(\sim\langle N \rangle^4\right)$$

$$\kappa_2 = \langle N \rangle + \left(\sim\langle N \rangle^2\right)$$

polynomial in Δy

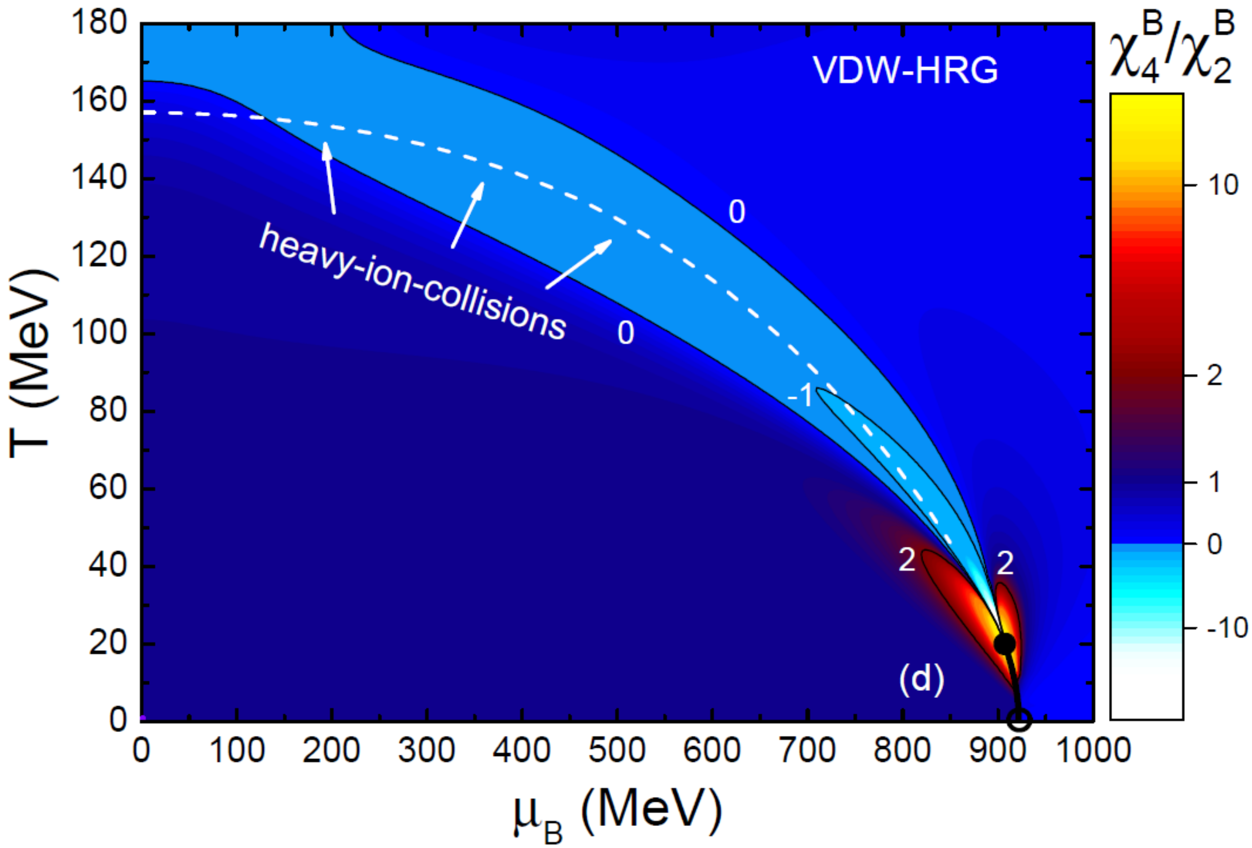
Cumulant ratios may strongly depend on acceptance in rapidity and in transverse momentum

Comparison with models which do not have experimental acceptance is questionable

Comparison with lattice QCD calculations is very tricky

With long-range rapidity correlations the cleanest observable is $\frac{C_i}{\langle N \rangle^i}$

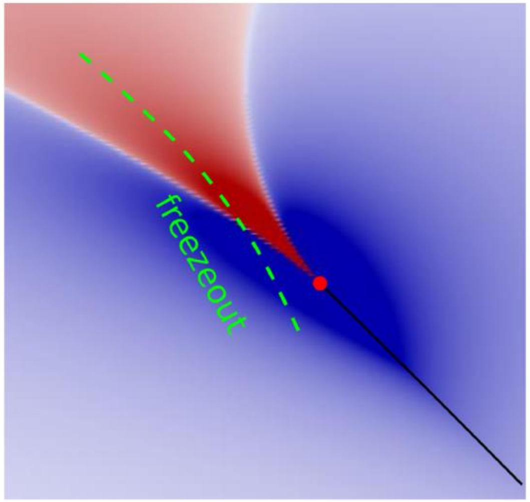
HRG with attractive and repulsive Van der Waals interactions between (anti)baryons



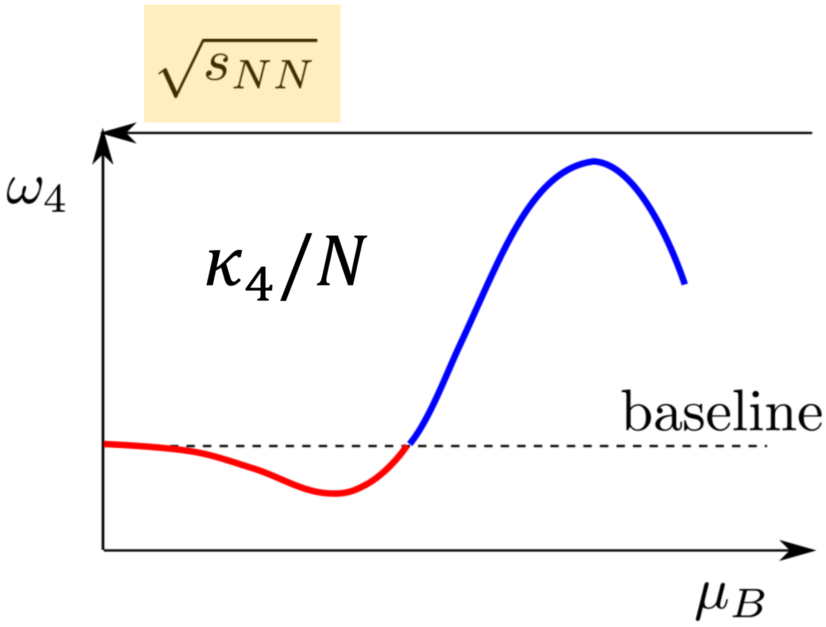
κ_4/κ_2

V. Vovchenko, M.I. Gorenstein, H. Stoecker, PRL 118 (2017) 182301

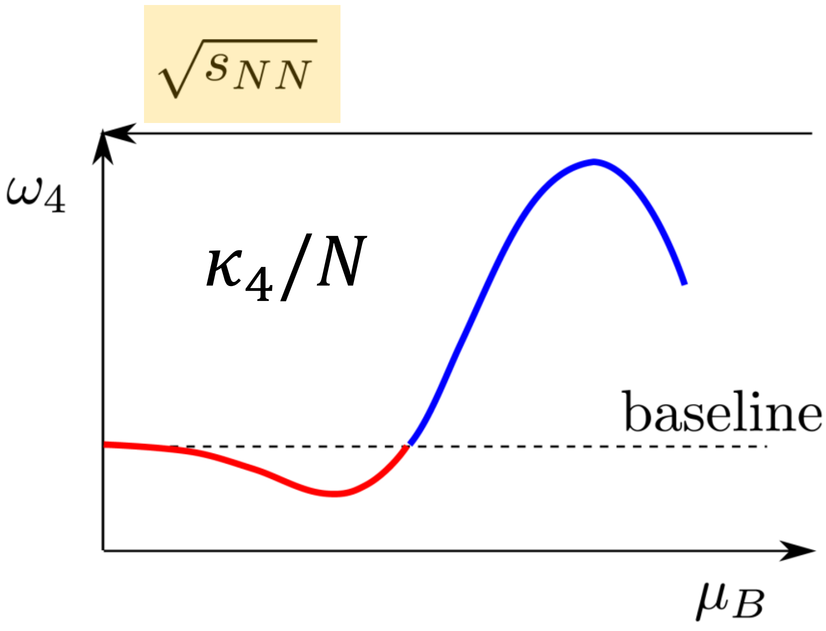
M.A. Stephanov PRL 107 (2011) 052301



Density plot of the quartic cumulant obtained by mapping the Ising model into QCD.
Freezeout line is for demonstration only.



Normalized quartic cumulant of proton multiplicity

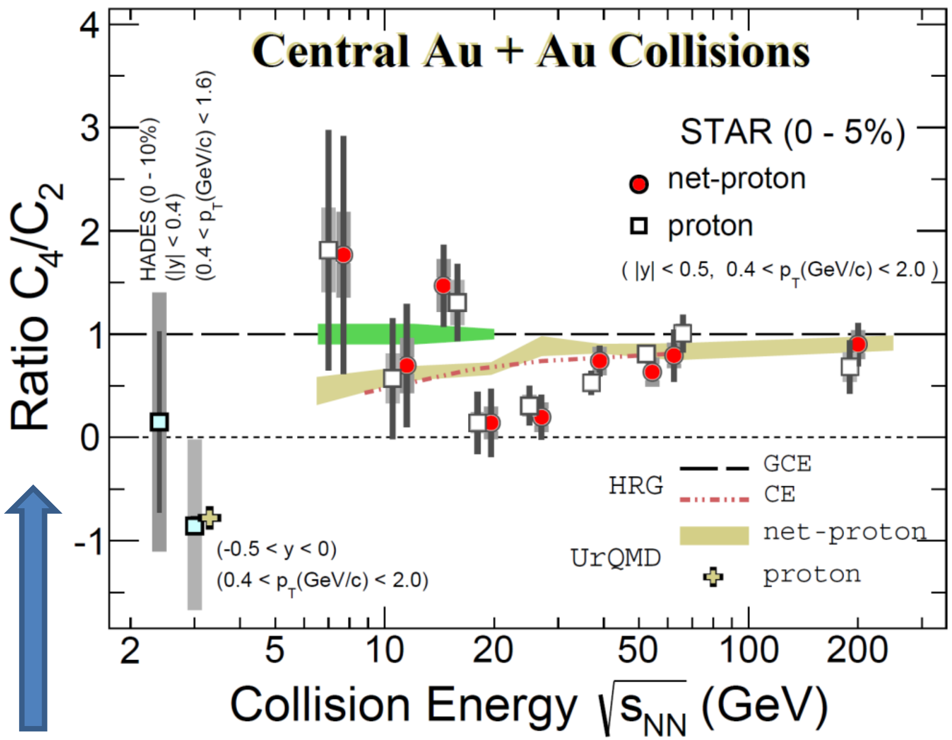


D. Mroczek, A.R. Nava Acuna, J. Noronha-Hostler, P. Parotto, C. Ratti, M.A. Stephanov, PRC 103 (2021) 3, 034901

We find that, while the peak remains a solid feature, the presence of the critical point does not necessarily cause a dip in χ_4^B *on the freezeout line below the transition temperature.*

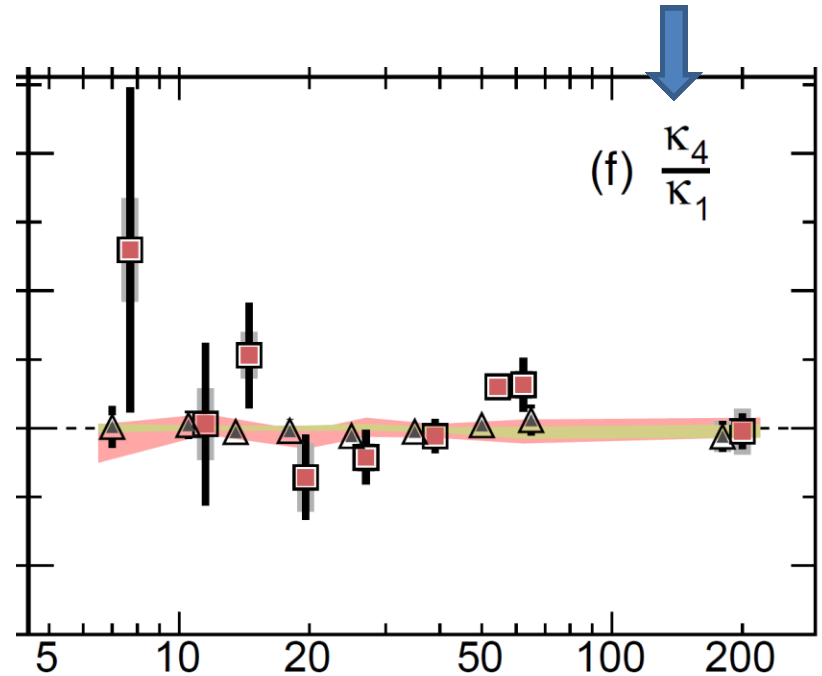
STAR Collaboration

2112.0024
PRC 104 (2021) 2, 024902



κ_4/κ_2

C_4/C_1

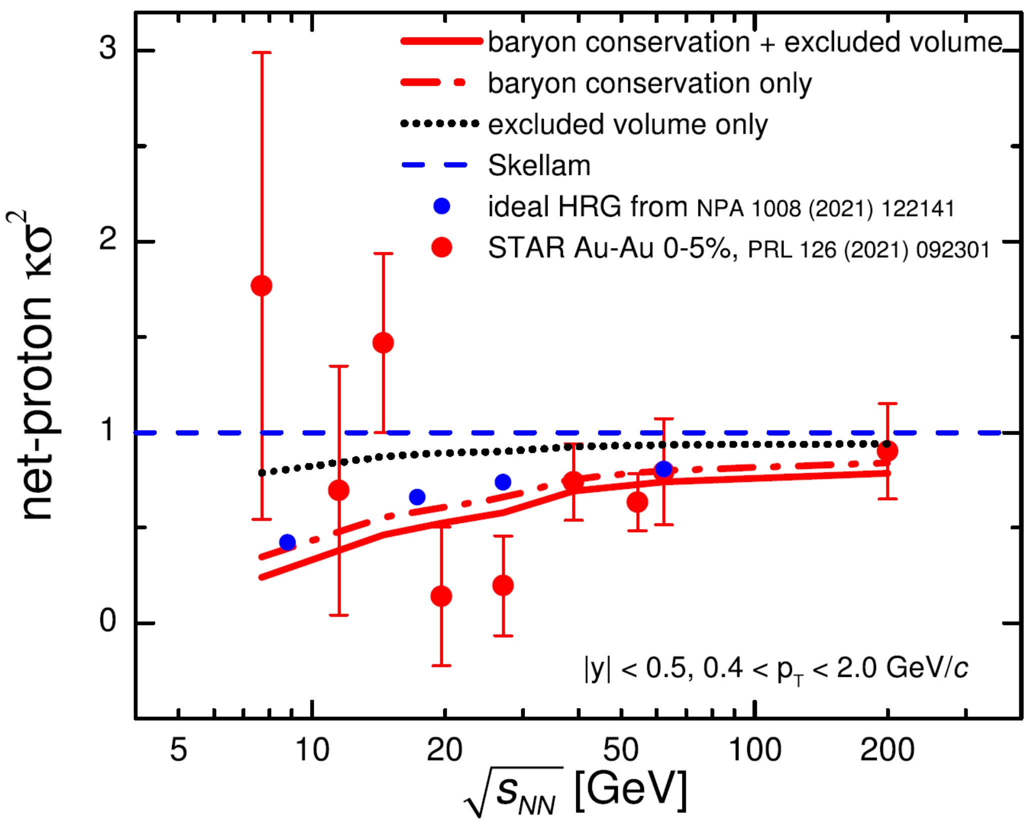


Visible four-proton correlations at 7.7 GeV (large errors)

A hint of non-monotonic dependence

STAR data vs. hydrodynamics with baryon conservation and excluded volume

κ_4/κ_2

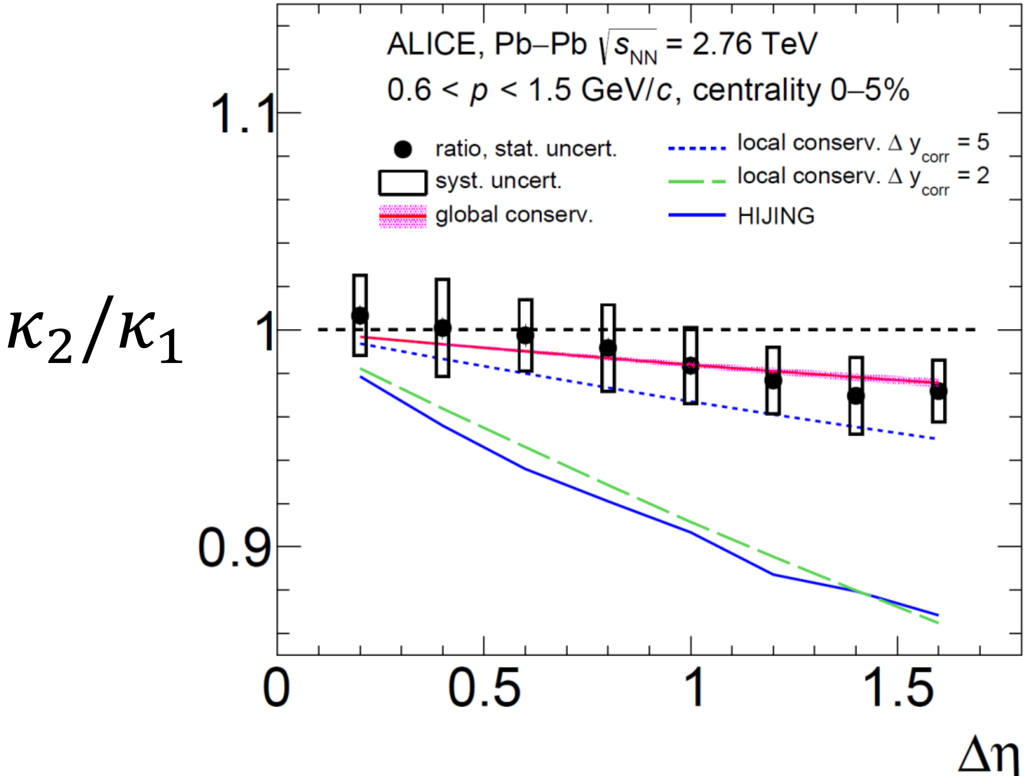


Baryon conservation for $\sqrt{s} > 20$ GeV

V.Vovchenko, V.Koch, C.Shen, PRC 105, 014904 (2022)
P.Braun-Munzinger, B.Friman, K.Redlich, A.Rustamov, J.Stachel, NPA 1008 (2021) 122141
AB, V.Koch, V.Skokov, PRC 87 (2013) 1, 014901

ALICE Collaboration

PLB 807 (2020) 135564

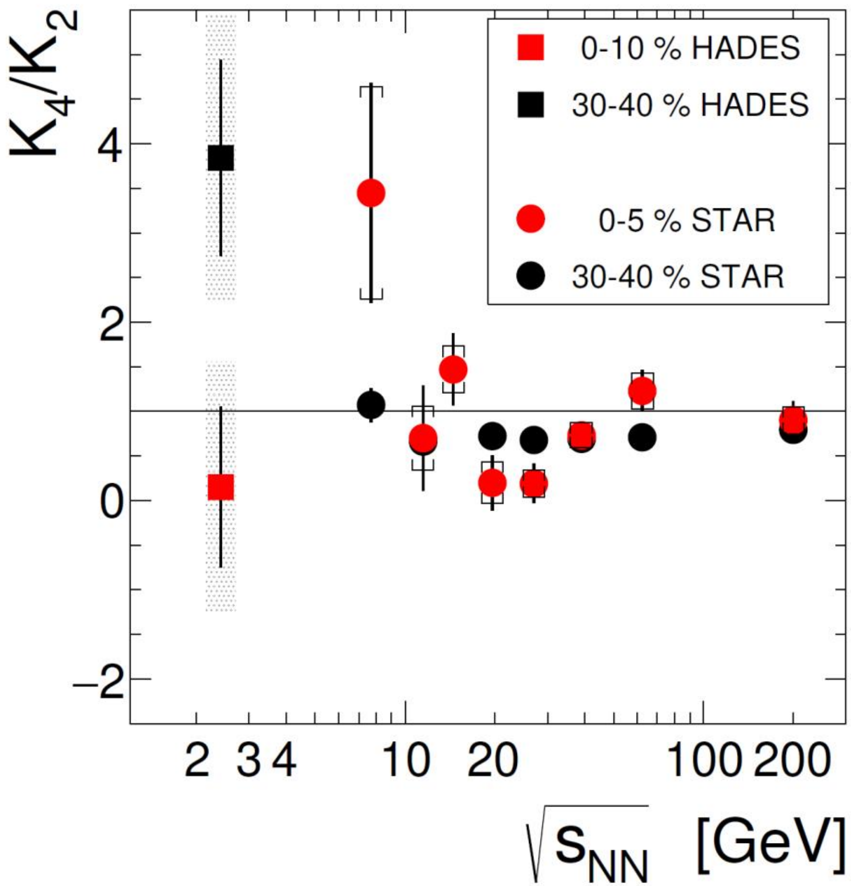


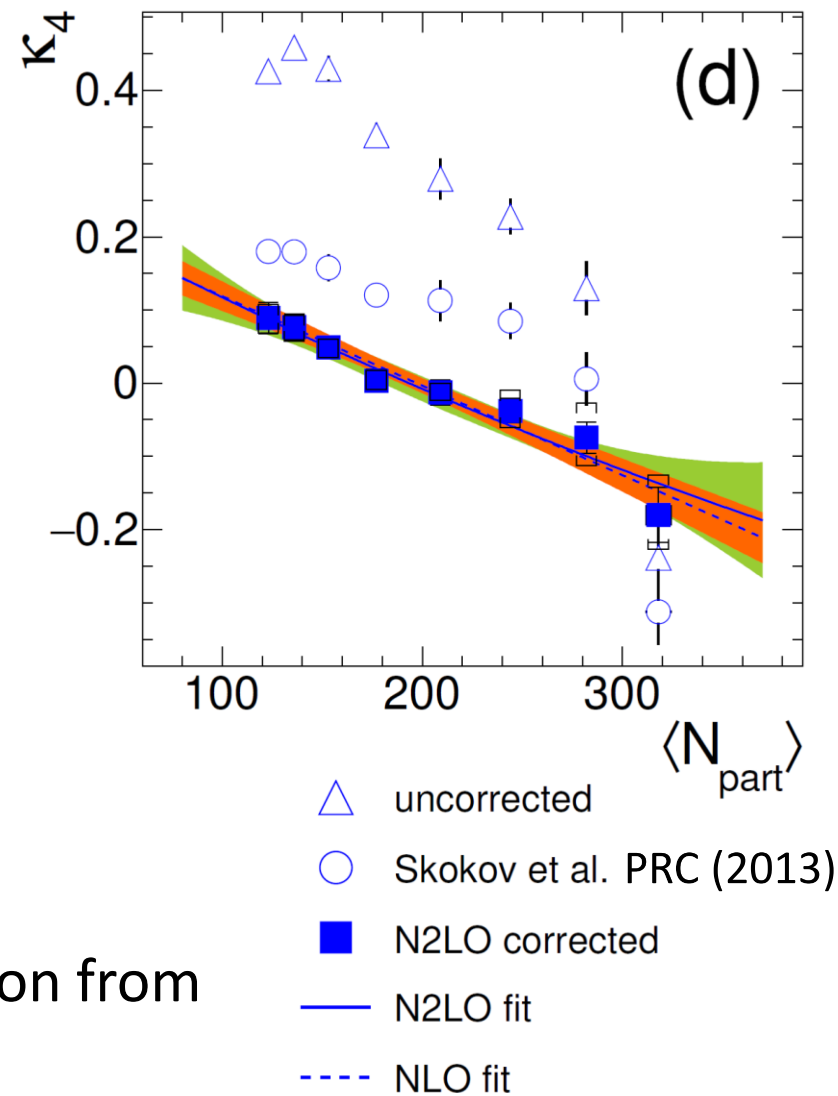
Global (not local) baryon conservation! Something to understand. It would be good to measure proton, antiproton and mixed proton-antiproton factorial cumulants M.Barej, AB, PRC 102 (2020) 6, 064908

See O.Savchuk, V.Vovchenko, V.Koch, J.Steinheimer, H.Stoecker, PLB 827 (2022) 136983
Local conservation and $B\bar{B}$ annihilation

HADES Collaboration

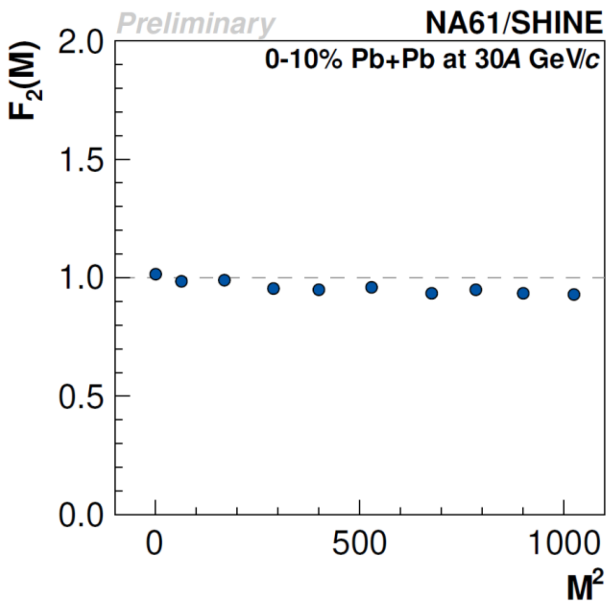
PRC 102 (2020) 2, 024914

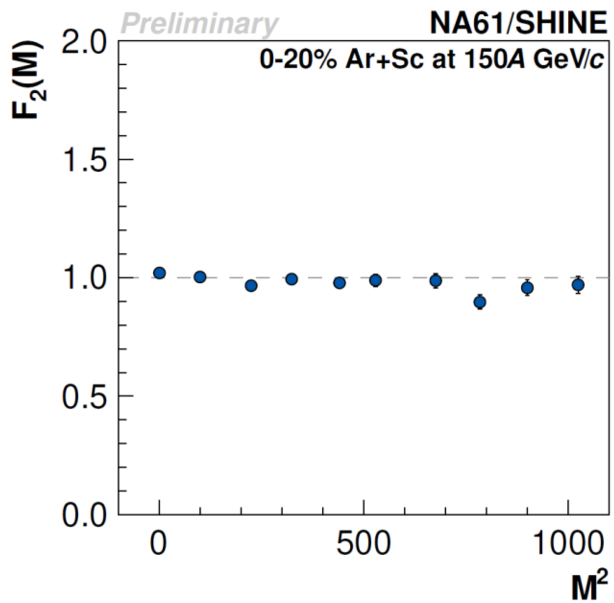




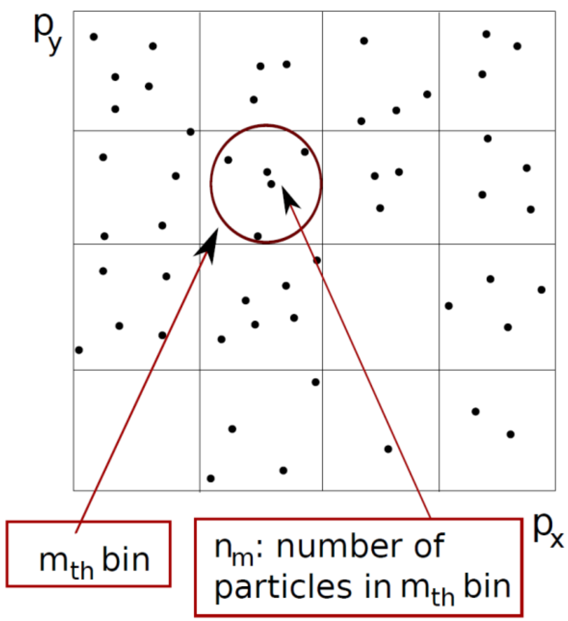
Significant correction from volume fluctuation

NA61/SHINE Collaboration





statistical uncertainties only

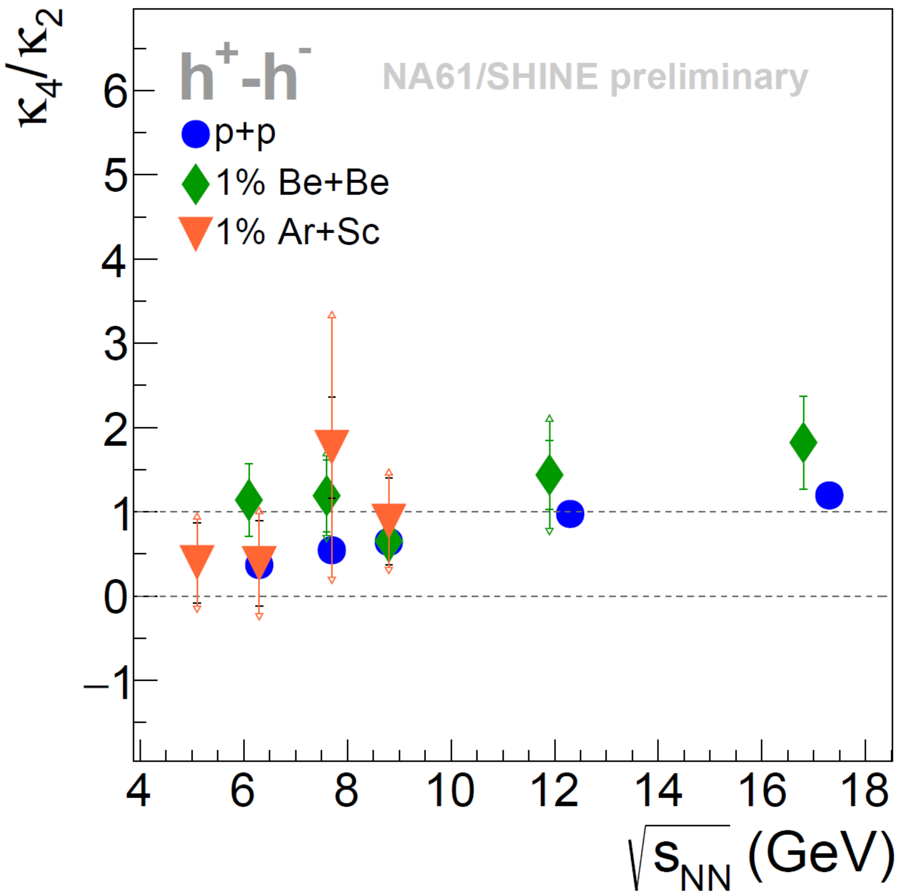


$$F_2(M) = \frac{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \left(n_i - 1\right) \right\rangle}{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \right\rangle^2}$$

$$F_2(M) \sim (M^2)^{5/6}$$

N.G. Antoniou, F.K. Diakonos, A.S. Kapoyannis, K.S. Kousouris, PRL 97, 032002 (2006)

NA61/SHINE Collaboration



No critical signal. Consistent with p+p.

Conclusions

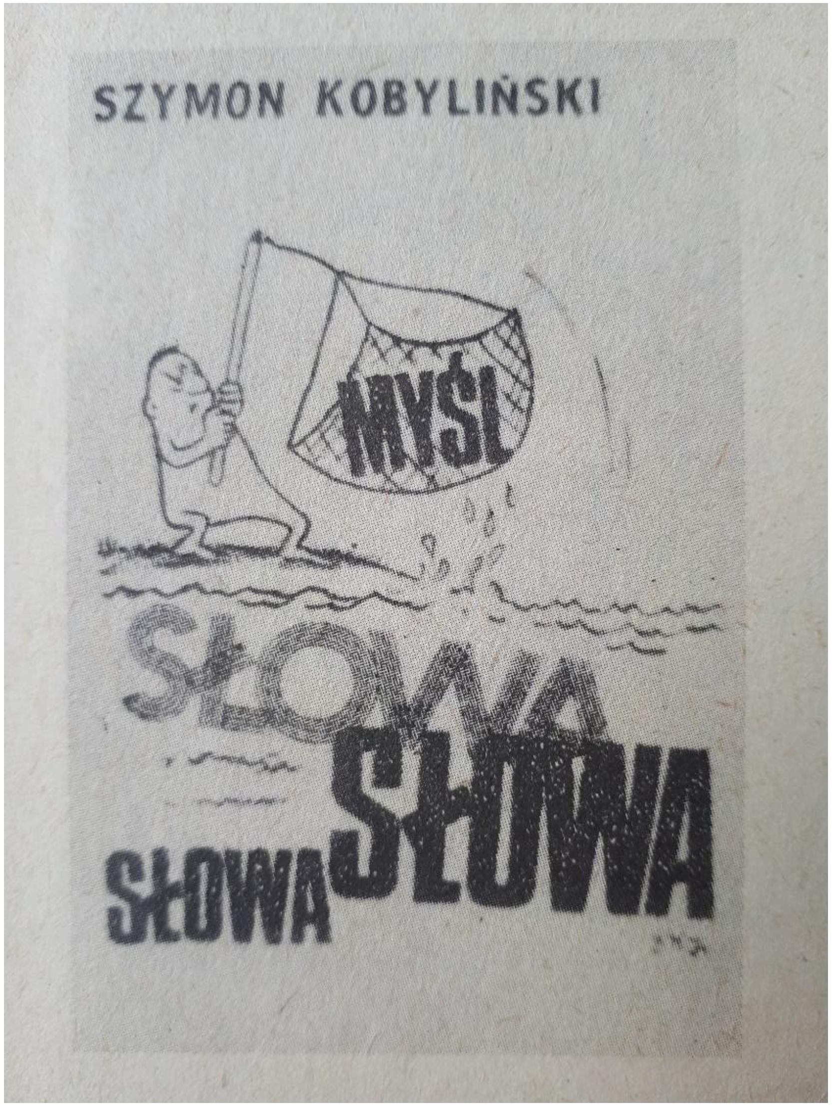
Interesting and important physics but so far no success

Clear signal of (global?) baryon conservation

Interesting STAR point at 7.7 GeV.
Four-proton correlations (physics?)

We definitely need better statistics

Hopefully some progress will come from lattice QCD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